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OPTIMAL CONTROL OF GREENHOUSE CLIMATE: COMPUTATION OF THE INFLUENCE OF FAST AND SLOW DYNAMICS

R.F. TAP*, L.G. VAN WILLIGENBURG*, G. VAN STRATEN* and E.J. VAN HENTEN**

** Wageningen Agricultural University, Department of Agricultural Engineering and Physics, agrotechnion, Bomenweg 4, 6703 HD Wageningen, The Netherlands*

*** Institute of Agricultural Engineering (IMAG), P.O. Box 43, 6700 AA Wageningen, The Netherlands*

Abstract. In case of optimal greenhouse climate control the fast greenhouse dynamics are generally ignored. Only the slow dynamics that describe the crop behaviour are considered. Through the computation of optimal climate controls for growing lettuce in greenhouses, subjected to actual weather, it is demonstrated that the neglect of the greenhouse dynamics seriously affects the result (net profit).

Key Words. Agriculture; greenhouse climate; multivariable control systems; optimal control; optimisation

1. INTRODUCTION

The profit obtained from lettuce crop production in greenhouses is mainly determined by the production rate, the price of lettuce, and the costs associated with maintaining a favourable climate in the greenhouse. The control of the temperature, the CO₂-concentration and the relative humidity, the main climate variables in the greenhouse, results in conflicting interests concerning heating, ventilation and CO₂ supply, the main control variables. At present hierarchical rules based on experience guide the choice of setpoints for temperature, CO₂-concentration and relative humidity. In reality the market gardener does not want to realize setpoints but he wants to maximise profit.

Optimal control of greenhouse climate entails the operation of the control variables such that the economic profit of the grower is maximised. The basis of this approach is a crop model that describes the crop behaviour under influence of the indoor climate conditions which in turn are determined by the outside weather conditions and the exerted control actions. This system is characterised by both fast and slow dynamics, the first associated with the greenhouse climate and the second with crop growth. The computation of optimal controls for such systems raises numerical difficulties (Kalman, 1964). In the literature solutions for the seasonal optimisation problem have been presented assuming slowly changing weather (e.g. Seginer, 1992; van Henten and Bontsema, 1991). As the dynamics describing crop-behaviour are much slower than the physical greenhouse dynamics, the latter can be ignored in this case and seasonal optimization can treat the physical climate as immediately realizable through the control.

It can be shown that the error by assuming the greenhouse dynamics to be infinitely fast in these type of calculations is small (van Henten and Bontsema, 1992). However, because the weather in reality changes fast and is a dominant disturbance, it is no longer obvious that the greenhouse dynamics can still be ignored. To compute optimal controls the weather should be completely known over the time interval over which the optimisation takes place. Since long term weather predictions are unreliable we are confronted with a trade off concerning the choice of this time interval. If for instance we take it small the weather predictions used will be good and we have no numerical difficulties, however the system behaviour in the long run is not taken into account.

Our aim is to determine the effects of neglecting the influence of the fast changing weather and the fast greenhouse dynamics on the calculated optimal control. To do this we will consider the optimisation over a short period of time (compared to the growing period) for the situation with and without greenhouse dynamics and by using either measured weather data or a smoothed version of it. It is assumed to be known in advance what the weather will be like every minute (perfect weather prediction). The criterion of comparison will be the profits made when the optimal controls resulting from each of these four combinations are applied to the system modeled by fast greenhouse dynamics subjected to the measured weather data.

2. OPTIMAL CONTROL PROBLEMS

The crop model describes the crop behaviour under

influence of the indoor climate conditions, which in turn are determined by the outside weather conditions and the exerted control actions. This can be formalised as

$$\frac{dx_c}{dt} = f(x_c, x_p, u_e, u_c) \quad (1)$$

$$\frac{dx_p}{dt} = g(x_c, x_p, u_e) \quad (2)$$

in which x_c are state variables that represent the indoor climate,

$$x_c = \begin{pmatrix} x_{c1} \\ x_{c2} \\ x_{c3} \\ x_{c4} \end{pmatrix} = \begin{pmatrix} T_g \\ T_s \\ C_i \\ V_i \end{pmatrix} \quad (3)$$

where T_g is the greenhouse temperature, T_s is the greenhouse soil temperature, C_i is the greenhouse CO₂ concentration and V_i is the greenhouse water vapour density. The variable x_p represents state variables associated with crop development,

$$x_p = \begin{pmatrix} x_{p1} \\ x_{p2} \end{pmatrix} = \begin{pmatrix} W_n \\ W_s \end{pmatrix} \quad (4)$$

where W_n is the non-structural dry weight and W_s is the structural dry weight of the crop. The external inputs u_e are given by

$$u_e = \begin{pmatrix} u_{e1} \\ u_{e2} \\ u_{e3} \\ u_{e4} \\ u_{e5} \end{pmatrix} = \begin{pmatrix} T_o \\ G \\ w \\ C_o \\ R_o \end{pmatrix} \quad (5)$$

where T_o is the outside air temperature, G is the incoming shortwave radiation, w is the wind speed, C_o is the outside air CO₂ concentration and R_o is the outside relative humidity. Finally the control u_c is defined by

$$u_c = \begin{pmatrix} u_{c1} \\ u_{c2} \\ u_{c3} \end{pmatrix} = \begin{pmatrix} H \\ r_w \\ \phi_i \end{pmatrix} \quad (6)$$

where H is the heat input, r_w is the relative window aperture and ϕ_i is the CO₂ injection flux. Given the model (1)-(5) the goal is to maximise the criterion,

$$J = \Phi(x_p(t_f)) - \int_0^{t_f} L(x_c, x_p, u_e, u_c) dt \quad (7)$$

which represents the profit made within $[0, t_f]$. The function Φ represents the benefits obtained from marketing the product at the final time t_f . The function L represents the cost made at any instant in time related to CO₂ injection and heating.

The problem simplifies when the indoor climate is considered so fast, that it is practically memory-less compared to crop growth. In that case f is considered to be zero and then x_c algebraically depends on u_e , u_c and x_p and the model can be formalised as

$$\frac{dx_p}{dt} = h(x_p, u_e, u_c) \quad (8)$$

To investigate the importance of considering the greenhouse dynamics results obtained with the model (1), (2) are compared with those of (8). The functions f (van Henten, 1993), g (Tchamitchian *et al.*, 1993) and J are described explicitly in the appendix. The function h can be derived from f and g .

For lettuce the function Φ equals the weight of the heads of lettuce times the price of the product per kilo. When an optimisation is carried out for one season the price is the auction price, which we consider to be known. The growth of the crop at different times during the season has different meaning for the final weight of the product. In order to perform an optimisation for a part of the season, the price must reflect the relative importance of that part of the season. Van Henten has shown that the course in time of this importance is almost independent of the weather. So the price used in short term optimisations can be determined on the basis of long term considerations. Because this has not been done yet, the price used here is only an estimation. Since our goal is to compare optimal solutions this is not a serious drawback.

3. COMPUTATION OF OPTIMAL CONTROLS AND COMPARISON

The relative humidity is not part of the crop growth model. It is believed however that the humidity should stay within certain borders. To enforce this we have introduced in L a penalty function P which punishes exceedings of those borders.

The optimal control is obtained by solving the associated two point boundary value problem (TPBVP) with a first order gradient algorithm. To discover the influence of the greenhouse dynamics this is done for both greenhouse model (1), (2) and greenhouse model (8). The influence of fast varying weather on the optimal control is determined by calculating the optimal control for both measured weather and a smoothed version of it. This provides four situations: dynamic greenhouse model (1), (2) and measured weather (dm), dynamic greenhouse model (1), (2) and smoothed weather (ds), static greenhouse model (8) and measured weather (sm) and static greenhouse model (8) and smoothed weather (ss).

The smoothed version of the weather is obtained by applying a moving average filter over a two hours

period to the measured weather data, where the result is assigned to the middlemost point of that period.

To make a relevant comparison we simulated the behaviour of the greenhouse model (1), (2) influenced by the measured weather and the optimal controls generated by the four different options. In table 1 the results are given. The criterion value is divided into the net profit without considering the humidity bounds and the "penalty" costs associated with violating the humidity bounds. The penalty function is such that the associated costs are small compared to the real costs while keeping the humidity reasonably within its bounds. So a trade off exists between exceedings of the humidity bounds and the profit made by doing so. The window opening and the heat input are largely determined by the humidity bounds. Without humidity bounds the heating is turned off while the windows are closed during daytime and opened at night to cool the greenhouse. With the humidity bounds the heat input and the window opening are completely different.

The influence of the fast greenhouse dynamics emerges from the comparison of Fig. 1 and 2. Both sm and dm try to provide a high CO₂-concentration when there is a lot of radiation, but only dm is successful. Because dm considers the greenhouse dynamics it starts to dose CO₂ about 20 min. before there is a radiation peak. Because sm does not consider the greenhouse dynamics it doses CO₂ at the moment there is a radiation peak. Table 1 shows the consequences on the profit of this. Because ss is based on smoothed weather the controls are slowly varying and ss yields a better result than sm. Ds does not consider the fast weather variations. That's why ds yields a poorer result than dm.

4. DISCUSSION

Optimal control yields the best results when the system behaviour is completely known in advance. This implies perfect knowledge of the state equations and of the disturbances. The optimality of the controls is heavily influenced by the accuracy of the greenhouse dynamics, the crop dynamics, and the weather predictions. This calls for an accurate dynamic model and good weather forecasts. In case of the crop dynamics for example a better knowledge of the influence of humidity on crop growth would improve the control and make the penalty function redundant.

From the differences between ds and ss it is evident that the greenhouse dynamics have a considerable influence on the optimal control and corresponding profit. This influence is bigger when the weather is less smooth (dm versus sm), because this causes a non smooth control which together with the fast changing weather activates the fast dynamics which

can then no longer be ignored. Therefore if, in the optimization, the fast dynamics are considered short term weather predictions, which are hard to obtain at present, are also needed. This need may open a new direction of research. In practice the possibility to obtain reliable short term weather predictions will determine if we will actually gain by taking into account the greenhouse dynamics.

Table 1 Optimisation results (f/m^2)

	Penalty	Profit	Criterion
ds	$-7.6 \cdot 10^{-3}$	$14.1 \cdot 10^{-3}$	$6.5 \cdot 10^{-3}$
dm	$-0.3 \cdot 10^{-3}$	$14.5 \cdot 10^{-3}$	$14.2 \cdot 10^{-3}$
ss	$-2.8 \cdot 10^{-3}$	$8.4 \cdot 10^{-3}$	$5.6 \cdot 10^{-3}$
sm	$-1.1 \cdot 10^{-3}$	$6.3 \cdot 10^{-3}$	$5.2 \cdot 10^{-3}$

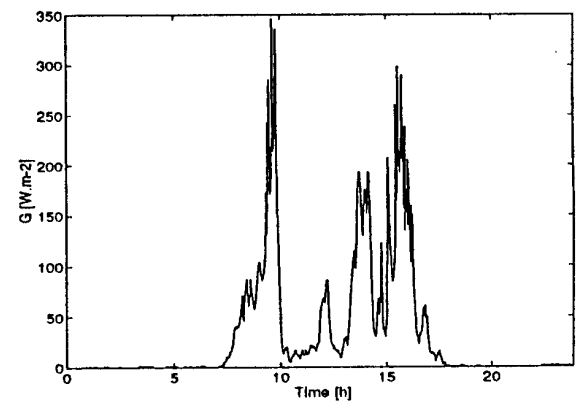


Fig. 1. Measured global radiation

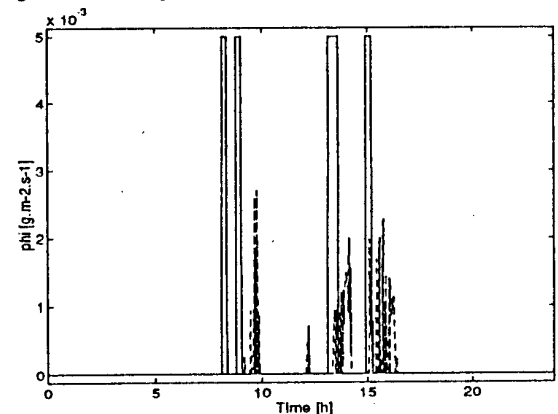


Fig. 2. Optimal CO₂ dosage — dm - - - - sm

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6. APPENDIX

6.1. Indoor Climate And Crop Model

The temperature dependence of g_x given by van Henten (1993), which is only valid for temperatures between 5 °C and 40 °C, has been modified to enlarge its domain of validity.

$$\frac{dT_g}{dt} = \frac{1}{C_g} [(k_v + k_r)(T_o - T_g) + H + k_s(T_s - T_g) + \eta G]$$

$$\frac{dT_s}{dt} = \frac{1}{C_s} [k_s(T_g - T_s) + k_d(T_d - T_s)]$$

$$\frac{dC_i}{dt} = \frac{A_g}{V_g} [\phi_v(C_o - C_i) + \phi_i - f_{gc} + f_{rc}]$$

$$\frac{dW_n}{dt} = \alpha f_{gc} - r_g W_s - f_{mr} - \frac{1-\beta}{\beta} r_g W_s$$

$$\frac{dW_s}{dt} = r_g W_s$$

$$\frac{dV_i}{dt} = \frac{A_g}{V_g} \left[A_p C_{pa} \left(\frac{a_v e^{\frac{17.4 T_p}{239 + T_p}}}{T_p + 273.15} - V_i \right) - \phi_v (V_i - V_o) \right]$$

$$k_v = M_a C_a \phi_v \quad \phi_v = \kappa + \lambda w + \gamma w r_w$$

$$R_p = t_g p G \quad A_p = 1 - e^{-K g_{sw} (1-\tau) W_s}$$

$$f_{gc} = A_p \frac{e R_p \mu \omega (C_i - \Gamma)}{e R_p + \mu \omega (C_i - \Gamma)} \quad e = e_0 \frac{C_i - \Gamma}{C_i + 2\Gamma}$$

$$\frac{1}{\mu} = \frac{1}{g_s} + \frac{1}{g_b} + \frac{1}{g_x} \quad g_x = \frac{g_{x1}}{(T_g + g_{x2})^2 + g_{x3}}$$

$$\Gamma = \Gamma_0 Q_\Gamma^{\frac{T_g - 20}{10}} \quad r_g = r_{gm} Q_g^{\frac{T_g - 20}{10}} \frac{W_n}{W_s + W_n}$$

$$f_{mr} = (r_s(1-\tau) W_s + r_r \tau W_s) Q_r^{\frac{T_g - 25}{10}}$$

$$f_{rc} = \frac{1}{\alpha} (f_{mr} + \frac{1-\beta}{\beta} r_g W_s)$$

$$T_p = \begin{cases} T_g & G \leq 10 \\ T_g + 2.5 & G > 10 \end{cases}$$

$$V_o = \frac{a_v R_o}{100(T_o + 273.15)} e^{\frac{17.4 T_o}{239 + T_o}}$$

$$P = \begin{cases} 10^{-7}(70 - R_i) & R_i \leq 70 \\ 0 & 70 < R_i < 90 \\ 10^{-7}(R_i - 90) & R_i \geq 90 \end{cases}$$

$$R_i = \frac{100}{a_v} V_i (T_g + 273.15) e^{\frac{-17.4 T_g}{239 + T_g}}$$

6.2. The Criterion

$$J = \alpha_1 (1-\tau) (W_n + W_s) - \int_0^{t_f} (\alpha_2 \phi_i + \alpha_3 H + P) dt$$

6.3. Model Parameters

$C_g = 32 \cdot 10^3 \text{ J.K}^{-1} \cdot \text{m}^2$	$k_r = 7.9 \text{ W.m}^2 \cdot \text{K}^{-1}$
$\eta = 0.7$	$C_s = 120 \cdot 10^3 \text{ J.K}^{-1} \cdot \text{m}^2$
$k_s = 5.75 \text{ W.m}^2 \cdot \text{K}^{-1}$	$k_d = 2.0 \text{ W.m}^2 \cdot \text{K}^{-1}$
$T_d = 10.0 \text{ }^\circ\text{C}$	$V_g/A_g = 3.0 \text{ m}$
$M_a = 1.29 \text{ kg.m}^3$	$C_a = 10^3 \text{ J.kg}^{-1} \cdot \text{K}^{-1}$
$\kappa = 5.03 \cdot 10^{-5} \text{ m.s}^{-1}$	$\lambda = 4.02 \cdot 10^5$
$\gamma = 3.68 \cdot 10^5$	$t_g = 0.55$
$p = 0.45$	$\alpha = 30/44$
$\beta = 0.8$	$K = 0.9$
$g_{sw} = 0.075 \text{ m}^2 \cdot \text{g}^{-1}$	$\tau = 0.07$
$\omega = 1.83 \cdot 10^{-3} \text{ g.m}^2 \cdot \text{ppm}^{-1}$	$e_0 = 17 \cdot 10^{-6} \text{ g.J}^{-2}$
$g_s = 0.005 \text{ s.m}^{-1}$	$g_b = 0.007 \text{ s.m}^{-1}$
$g_{x1} = 0.3 \text{ s.K}^2 \cdot \text{m}^{-1}$	$g_{x2} = -24 \text{ K}$
$g_{x3} = 75 \text{ K}^2$	$r_{gm} = 5.8 \cdot 10^6 \text{ s}^{-1}$
$Q_g = 1.6$	$\Gamma_0 = 40 \text{ ppm}$
$Q_r = 2.0$	$r_s = 3.47 \cdot 10^7 \text{ s}^{-1}$
$r_r = 1.16 \cdot 10^{-7} \text{ s}^{-1}$	$Q_r = 2.0$
$C_{pa} = 3.53 \cdot 10^{-3} \text{ m.s}^{-1}$	$a_v = 1.32 \text{ kg.m}^3$
$\alpha_1 = 0.02 \text{ f.g}^{-1}$	$\alpha_2 = 1.2 \cdot 10^{-4} \text{ f.g}^{-1}$
$\alpha_3 = 7.3 \cdot 10^{-9} \text{ f.W}^{-1}$	

6.4. CO₂ Conversion Factor

$$1 \frac{\text{g}}{\text{m}^3} = \frac{8314(T+273.15)}{44 \cdot 10^{-3} \times 1 \cdot 10^{-6} \times 1.01 \cdot 10^5} \text{ ppm}$$