DIGITAL OPTIMAL REDUCED-ORDER CONTROL OF A SOLAR POWER PLANT

L.G. Van Willigenburg, J. Bontsema
Systems and Control Group
Wageningen University
Technotron, Bornsesteeg 59
6708 PD Wageningen
The Netherlands.
Tel.: 31317482941, Fax: 31317484819
E-mail: gerard.vanwilligenburg@wur.nl

W.L. De Koning
Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology
E-mail: w.l.dekoning@ewi.tudelft.nl

L. Valenzuela, C. Martinez
Plataforma Solar de Almeria-CIEMAT
Tabernas (Almeria), Spain
E-mail:loreto.Valenzuela@psa.es

ABSTRACT

So far, researchers have developed *simplified* models to control the so called Acurex field, which is a solar power plant situated in Tabernas in Spain, used mainly for research purposes. Most of these simplified models for control have been used in simulation. Some have also been used in control experiments. Usually an *advanced* model of the plant was used to identify and verify the simplified model. This advanced model is a finite dimensional approximation of the *physical description* of the plant by two partial differential equations. In this paper we use the advanced model *itself* to control the plant. Despite the high dimension of the model our digital optimal reduced-order controller design procedure enables us to synthesize a digital optimal reduced-order controller with just one state variable. This controller is shown to work properly in simulation and also in a control experiment.

1. INTRODUCTION

When *simplified* or *reduced-order models* are developed for control system design, in general, optimality is lost and stability may be lost. A major advantage of digital optimal reduced-order controller synthesis is that no simplified or reduced-order models need to be developed, but instead, possibly high dimensional models describing accurately the physics of a plant may be used directly for digital optimal reduced-order control system design. Having researched and developed algorithms for this type of control system design [1-5] we want to investigate their applicability and performance in industrial applications.

The Acurex field, a solar power plant in Tabernas in Spain, developed mainly for research purposes, turned out to be well suited for this. The plant is physically described by two partial differential equations and therefore an accurate physical description of the plant requires a high dimensional model. Furthermore such a model has been developed already [6,7]. Our objective therefore was to apply our digital optimal control and digital optimal reduced-order LQG compensator algorithms directly to this model, to design a digital optimal reduced-order controller for the solar power plant.

Although the high dimensional model approximating the physical description of the plant had been developed [6,7], not all the details of the model, needed for computation, have been published. In section 2 we describe our version of this model, including all the details needed for computation. One parameter of the model was tuned by hand, based on data obtained from several runs of the plant, the other parameter values were all obtained from [6,7] and some private communication.

In section 3 we present our digital optimal control system design procedure and mention briefly the algorithms required for the digital optimal reduced-order controller synthesis, the details of which can be found in the sited references. There is one major difficulty with the synthesis of our digital optimal reduced-order controller. Opposite to the situation considered in [1-5] our system has external inputs and therefore is described by,

$$\dot{x} = f\left(x,u,d\right) \tag{1.1}$$

where $x \in R^{n_x}$ denotes the state vector, $u \in R^{n_u}$ the control input vector, and $d \in R^{n_d}$ the external input vector. Due to the presence of external inputs, to synthesize our controller, we need to make a prediction in advance of the time response $d\left(t\right)$ of the external inputs. This prediction we will denote by $d^*\left(t\right)$. In our system two external inputs are solar radiation and ambient temperature. Unfortunately, in general, these cannot be predicted accurately in advance. On the other hand both solar radiation and outside temperature are measured accurately on-line. This can be described by the following output equation,

$$y = g\left(x,d\right) \tag{1.2}$$

where $y \in R^{n_y}$ are measurements which may depend explicitly on both the state and the external inputs. A procedure is presented in section 3 of this paper which incorporates in our digital optimal reduced-order controller design both predictions $d^*\left(t\right)$ of the time response of external inputs as well as the use of on-line measurements, that may depend explicitly on both the state and the external inputs.

In section 4 we finally present and discuss simulation and experimental results.

2. PHYSICAL DESCRIPTION OF THE SOLAR POWER PLANT

For detailed information concerning the set-up and physical properties of the solar power plant we refer to [6,7]. Roughly, oil is pumped through pipes which together form ten parallel loops. Roughly, each parallel loop behaves identical. Mirrors around the pipes are focussed on the pipes so that the oil heats up due to the reflected solar radiation. The oil is stored in, and is withdrawn from, a stratification tank from which heat and therefore energy can also be extracted. Our model of the plant describes the heat transport phenomena which occur in one single loop of the solar power plant. Our digital optimal reduced-order controller is designed to control the oil flow through this single loop. If we make the assumptions described in [6,7] the heat transport phenomena in the single loop are described by the following two partial differential equations.

$$\rho_m c_m A_m \frac{\partial T_m}{\partial t} = I\eta_0 D - H_1 G\left(T_m - T_a\right) - LH_t\left(T_m - T_f\right) \quad (1.3)$$

$$\rho_f c_f A_f \frac{\partial T_f}{\partial t} = LH_t\left(T_m - T_f\right) - \rho_f c_f \dot{V}\frac{\partial T_f}{\partial x} \quad (1.4)$$

where ρ denotes density, c heat capacity, A transversal area, T temperature, I irradiance, H_1 thermal loss coefficient, D mirror width, H_t metal fluid heat transfer coefficient, G exterior diameter of the pipe, L inner diameter of the pipe, $\dot{V}$ oil flow (single control input), η_0 optical efficiency, x position along the pipe. The subscript m denotes metal (of the pipe), f fluid (the oil), a ambient. The following additional relations apply, some of which are obtained from polynomial fits of data,

$$\rho_f = 903 - 0.672\,T_f \left[kg\ m^{-3}\right],$$
$$c_f = 1820 + 3.478\,T_f \left[J\ kg^{-1}\ {}^{o}C\right]$$
$$H_t = \dot{V}^{0.8}\left(2.17e6 - 5.01e4\,T_f + 4.53e2\,T_f^2 - 1.64\,T_f^3 + 2.1e-3\,T_f^4\right)$$
$$H_1 = 0.00249\,T_m - 0.006133$$
$$I = \left(1 - \cos^2\left(\frac{\pi}{180}\delta_l\right)\sin^2\left(\frac{\pi}{180}\delta_2\right)\right)I_s$$
$$\delta_l = 23.45\sin\left(2\pi\left(284 + JD\right)/365\right),\ \delta_2 = 15\left(h_s - 12\right)$$

where $I_s\left[Wm^{-2}\right]$ denotes solar radiation, JD Julian Day (day number of the year 1-365) and h_s solar hour (0-24). The parameter values are as follows. $\eta_0 = 0.285$, $G = 0.0318\left[m\right]$, $L = 0.0276\left[m\right]$, $D = 1.82\left[m\right]$. The value of η_0 differs significantly from the ones reported in [5,6] and was fitted manually on several different data sets.

For computation a finite difference approximation, the details of which are specified in section 3.1, was used to approximate the model (1.3), (1.4).

3. DIGITAL OPTIMAL REDCUCED-ORDER CONTROLLER SYNTHESIS

The synthesis of the digital optimal reduced-order controller is along the lines described in [1-4] and is represented by figure 1.

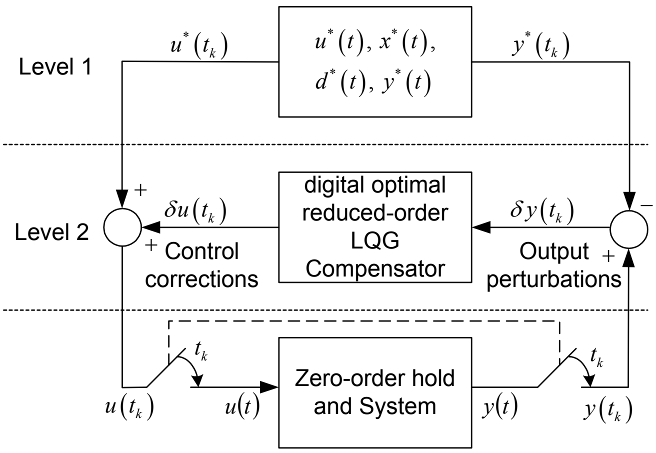

Figure 1: The digital optimal reduced-order control system

The design takes place at two levels. At the first level, based on the full non-linear model representing the finite-difference approximation, the associated initial state, and a to be minimized cost function, a digital optimal control,

$$u^*(t) = u^*(t_k),\ k = 0,1,..,N-1 \quad (1.5)$$

is computed where t_k, $k = 0,1,..,N-1$ are the sampling instants and t_N is the final time. The associated state-trajectory $x^*(t)$ and output trajectory $y^*(t)$ are also computed. This computation is performed off-line. The algorithm used is the one described in [2]. In our case however the sampling occurs conventionally, i.e. synchronously, with a fixed sampling interval of 60 seconds,

$$t_{k+1} - t_k = T_s = 60\ [\mathrm{s}],\ k = 0,1,..,N-1 \quad (1.6)$$

Furthermore at the first level we require the predicted time responses $d^*(t)$ of the external inputs. At the second level, based on the linearised dynamics about the digital optimal control and state trajectory computed at the first level, a quadratic cost function, and a description of the model and measurement uncertainties by small additive white noise, a digital optimal reduced-order LQG compensator is computed off-line. This compensator is used to attenuate on-line errors. The computation of this compensator concerns solving a time-varying finite horizon equivalent discrete-time optimal reduced-order LQG problem. A new algorithm, recently proposed in [8], is used to perform this computation. This algorithm is more efficient than others, especially if the prescribed dimension of the digital optimal LQG compensator, one in our case, is significantly smaller than that of the system, thirty five in our case.

3.1 Design specifications and computations at level 1

The cost function represents the desire to minimize the energy loss to the environment and to achieve a sufficiently high outlet temperature of the oil to not destroy the stratification of the oil in the tank. This temperature is approximately 220 degrees Celsius and therefore the cost function reads,

$$J\left(\dot{V}\left(t\right)\right)=1000\left(T_f^{out}-220\right)^2+\int_{t_0}^{t_N}\int_0^x H_1 G\left(T_m-T_a\right)\,dx\,dt \tag{1.7}$$

where T_f^{out} denotes the temperature of the oil flowing out of the metal pipe at the end, i.e. at $x=142$ m. Furthermore $t_0=11.7189$ [solar hour] and $t_N=14.7189$ [solar hour]. Then from (1.6) $N=180$. The number 1000 is a weighting factor. Initially we assume that both the metal and fluid (oil) have the same temperature at every location equal to $148\,^{o}C$. This assumption is based on the temperature of the fluid (the oil) at the bottom of the stratification tank which is about this value at the start of our control experiment. The initial temperature of the metal pipe, in reality, is different, depending on the solar radiation. Roughly therefore, during the first 10 to 20 minutes, the model and system behaviour will be different. The oil flow, which is the single control input, is bounded from above and below,

$$0.002\le\dot{V}\le0.012 \tag{1.8}$$

The upper bound is due to the maximum pump capacity and the lower bound is for safety, to prevent overheating of the oil. The predicted time response of the solar radiation and ambient temperature can be obtained by averaging previous responses, by taking the time response of the day before, by taking data from weather forecasts etc., or by making a suitable combination of these. We simply took the time response of the previous day. Finally for the incoming oil temperature response, which usually doesn't change much during a control experiment, we took the fixed value that was measured some time before the start of the experiment. Obviously these predictions are not very accurate but they are partly compensated for by the compensator which uses the measurements of the external inputs to on-line attenuate errors.

To compute the digital optimal control a finite difference approximation of the model (1.3), (1.4) is used that computes the temperatures at 16 nodes equally spaced over the 142 m. length of the pipe i.e. starting at $x=\Delta x=142/16$ m. and ending at $x=16\Delta x=142$ m. At $x=0$ the boundary condition is the fluid (oil) temperature flowing in, denoted by T_f^{in}, which acts as an external input and is also measured, like the solar radiation and ambient temperature. Figure 2 shows the computed optimal costs against the number of nodes which are all equally spaced over the metal pipe. From this figure it follows that 16 nodes is a suitable choice.

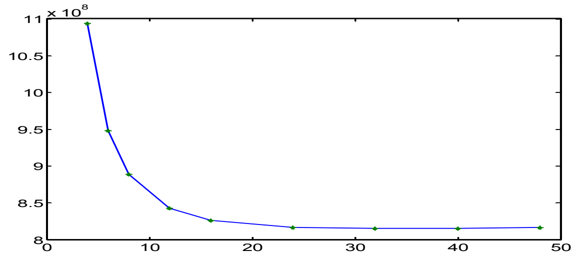

Figure 2: Computed costs versus the number of nodes

3.2 Design specifications and computations at level 2

To be able to use the on-line measurements, described by equation (1.2), for compensation, the linearised model describing the dynamic behaviour of perturbations, requires a *dynamic description (model)* of the behaviour of external input perturbations. Note that at level 1 only predictions $d^*\left(t\right)$ of the time-responses of the external inputs were used. These may be obtained from a dynamic model of the external inputs, but due to the uncertainty about these dynamics, we preferred to use other sources to obtain these time responses at level 1. Because the knowledge about the dynamics of the external inputs is poor and because, for compensation, the short term dynamic behaviour is of primary importance, at level 2 the dynamic behaviour of external input perturbations, denoted by δd, is described by,

$$\frac{d}{dt}\delta d=0 \tag{1.9}$$

In the linearised model describing the dynamics of perturbations, the external input perturbations, dynamically described by (1.9), now turn into additional state perturbations. So the augmented state, denoted by $\delta x'$, of the linearised model is given by,

$$\delta x'=\begin{bmatrix}\delta x\\ \delta d\end{bmatrix} \tag{1.10}$$

where δx is the vector of state perturbations and,

$$\delta d=\left[\delta I_s \quad \delta T_a \quad \delta T_f^{in}\right]^T \tag{1.11}$$

the vector of external input perturbations. Let * be the shorthand for $u^*\left(t\right)$, $x^*\left(t\right)$, $d^*\left(t\right)$, the digital optimal control, the associated optimal state trajectory and the prediction of the external inputs respectively, obtained from the design at level 1. Then the linearised system equals,

$$\frac{d}{dt}\delta x'=\begin{pmatrix}\left.\frac{\partial f}{\partial x}\right|_* & \left.\frac{\partial f}{\partial d}\right|_*\\ 0 & 0\end{pmatrix}\delta x'+\left.\frac{\partial f}{\partial u}\right|_*\delta u \tag{1.12}$$

$$\delta y=\begin{pmatrix}\left.\frac{\partial g}{\partial x}\right|_* & \left.\frac{\partial g}{\partial d}\right|_*\end{pmatrix}\delta x' \tag{1.13}$$

The linearised system (1.12),(1.13) is computed automatically through numerical differentiation using finite differences.

The linearised model uncertainties (errors) are represented by additive continuous-time white noise, the intensity matrix $V(t)$ of which is selected as follows,

$$V(t)=\frac{1}{\Delta x}\,diag\begin{pmatrix}100 & .. & 100 & 10^4 & 25 & 100\end{pmatrix} \qquad (1.14)$$

$$t_0 \le t \le t_N$$

This selection was based on [9] and the assumption that the oil and metal temperature errors, at each node of the pipe, are independent and that the spread due to these errors is everywhere equal to $\sqrt{100}$ =10°C. Similar arguments apply to the spread of the external inputs which were assumed to be $\sqrt{10^4}=100$ Wm^{-2}, $\sqrt{25}=5$ °C, $\sqrt{100}=10$ °C respectively.

Additional specifications needed for the digital optimal reduced-order LQG compensator concern the initial state perturbation $\delta x'(t_s)$, which were selected as follows,

$$E\left(\delta x'(t_s)\right)=0,\ \operatorname{cov}\left(\delta x'(t_s)\right)= \frac{1}{\Delta x}diag\begin{pmatrix}100 & .. & 100 & 10^4 & 25 & 100\end{pmatrix} \qquad (1.15)$$

These choices reflect the assumption that we have no a-priory knowledge about the initial perturbations, so their average values are set to zero. Furthermore the assumptions are equal to the ones used to obtain equation (1.14).

At the sampling instants (1.6) besides $d=\begin{bmatrix}I_s & T_a & T_f^{in}\end{bmatrix}^T$ also T_f^{out} is measured. Therefore in equation (1.2),

$$g(x,d)=\begin{bmatrix}T_f^{out} & I_s & T_a & T_f^{in}\end{bmatrix}^T \qquad (1.16)$$

The measurement uncertainty (errors) are represented by discrete-time additive white noise the covariance matrix of which is selected as follows,

$$W(t_k)=diag\begin{pmatrix}1 & 100 & 1 & 1\end{pmatrix},\ k=0,1,..,179 \qquad (1.17)$$

The selection (1.17) is obtained from the assumption that all measurement errors are independent, all temperature measurements having a spread of 1°C and the measurement of the solar radiation having a spread of $\sqrt{100}=10$ Wm^{-2}. To determine on-line all output perturbations $\delta y(t_k)$, the values of $y(t_k)$, given by (1.16) and (1.6), must be stored where I_s, T_a, T_f^{in} in (1.16) are obtained from the predicted time responses $d^*(t)$ used at level 1.

In the quadratic cost function for the digital optimal reduced-order LQG compensator design,

$$J(\delta u)=\delta x'^T(t_f)H\delta x'(t_f) + \int_{t_s}^{t_f}\delta x'^T(t)Q(t)\delta x'(t)+\delta u^T(t)R(t)\delta u(t)\,dt \qquad (1.18)$$

the design matrices $H, Q(t), R(t)$ where chosen as follows,

$$Q(t)=\Delta x\ diag\begin{pmatrix}1 & .. & 1 & 100 & 0 & 0 & 0\end{pmatrix},$$
$$R(t)=10^{10},\ t_s \le t \le t_f,\ H=0 \qquad (1.19)$$

The choice for $Q(t)$ penalises all state deviations quadratically by 1°C^{2}m^{-1},except for the oil outlet temperature section the penalty of which is selected to be 100°C^{2}m^{-1}. No penalties are associated with the external input perturbations, since these are uncontrollable, yet there measurements are used for compensation. The choice of $R(t)$ is such that the control corrections $\delta u(t)$ remain within the range (1.8). Finally the choice $H=0$ was made to prevent additional state perturbation penalties at the final time.

A digital optimal reduced-order LQG compensator with a state dimension of only one was computed for the solar power plant. The reasons for this were three fold. First of all, from the point of view of controller reduction, this choice is the most challenging one. Secondly, from figure 3 which shows the costs associated to each digital optimal reduced-order LQG compensator as a function of the state dimension of the compensator, it follows that reducing the state dimension to one, results in a loss of performance of only 0.8%.

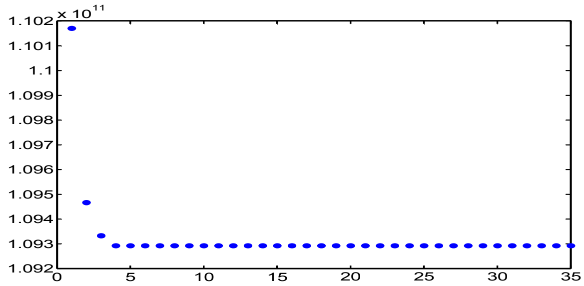

Figure 3: Digital optimal LQG costs versus the dimension of the compensator state

Finally the control software of the solar power plant, which runs on a Pentium II PC, does not permit the reading of data files, during the actual control of the plant. Since our controller is time-varying this presented a serious problem. The problem could only be resolved by implementing the time-varying compensator matrices as data into the *source code* of the control program. This was done by a simple code generator that turned all the matrix entries into text which was pasted into the source code of the control program in a suitable manner. This procedure *forced* us to select a state dimension of only one because, with a larger dimension, the *compiler*, which compiles the source code of the control program, ran out of memory, due to the large number of data in the source code.

4. SIMULATION AND EXPERIMENTAL RESULTS

The cost function (1.7) and the matrices (1.14), (1.15), (1.19), which determine the digital optimal reduced-order controller design, were selected based on several simulation experiments. To limit space we restrict ourselves to presenting the results, in figures 4 and 5, of one simulation of the digital optimal reduced-order control system that came out of the simulation experiments with external input data $d^*(t)$ of a day with a rather *constant* and *high* solar radiation (10-9-2003). The simulation concerns a day where the solar radiation was varying heavily, which should be considered as a worst case situation (15-9-2003).

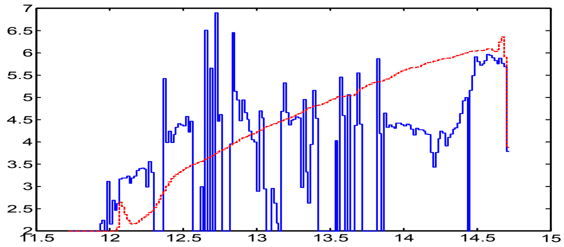

Figure 4: Optimal -- and simulated - oil flow dm^3 s^{-1}

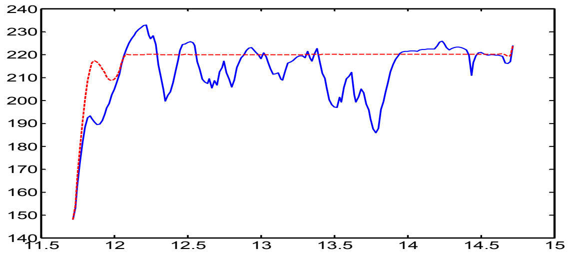

Figure 5:Optimal--and simulated - oil outlet temperature ^{o}C

From figure 4 and 5 observe that despite the large variations in the solar radiation the controller succeeds, whenever possible, in staying close to the desired 220 ^{o}C.

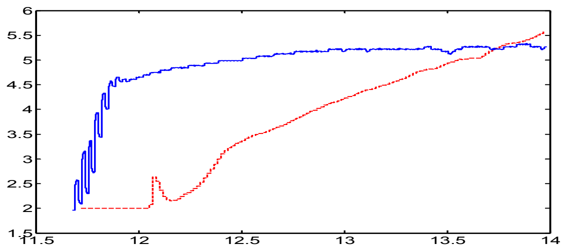

Figure 6: Optimal -- and actual - oil flow dm^3 s^{-1}

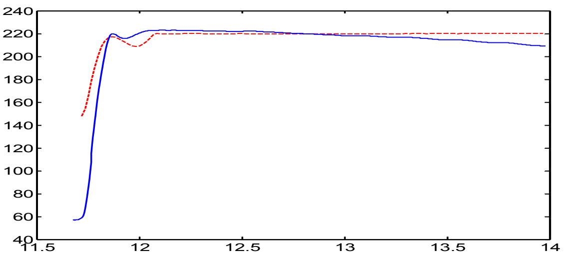

Figure 7: Optimal--and actual - oil outlet temperature ^{o}C

During our two weeks visit to Tabernas we were able to conduct one experiment on 19-9-2003 using the digital optimal controller tested in simulation. The results are presented in figures 6 and 7.

5. CONCLUSIONS

Except for one aspect, the solar power plant, as stated in the introduction, is suitable for testing our digital optimal reduced-order controller synthesis. This aspect concerns the uncertainty regarding the external inputs, which is generally high, and which may seriously deteriorate our digital optimal control system performance [1]. Despite this fact, the digital optimal control system performance, in worst case simulation and in the single experiment, was acceptable if not satisfactory. In conclusion we think that we have successfully tried our digital optimal reduced-order controller synthesis on the solar power plant, which may be considered an industrial application. The uncertainty in the external inputs of the plant suggests the use of a digital optimal receding horizon or model predictive controller. Such a digital controller however, requires much more on-line computation, but is expected to improve the on-line adaptation to the uncertain external inputs. Given the limitations of the current digital control equipment, which is scheduled to be innovated, the implementation of such a controller will only be possible in the near future.

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