



# Optimal control applied to tomato crop production in a greenhouse.

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## Abstract.

Present day climate control in greenhouses is realized by setpoint controllers, the setpoints being determined by hierarchical rules and/or growers experience. The choice of appropriate setpoints involves a lot of conflicting interests. Furthermore most setpoint trajectories cannot be realized in practice because of system constraints. Optimal control allows us to use explicit knowledge about the system behavior to determine an optimal compromise between conflicting interests in climate control. In our case the optimal compromise constitutes the maximization of profit from growing tomatoes in a greenhouse. Based on a simplified model which describes the growing of tomatoes in a greenhouse and the costs associated with climate control, we will compute optimal controls, i.e. the heating, carbon dioxide supply and ventilation of the greenhouse, using a first order strong variations algorithm, in a first attempt to improve the climate control. The results as well as problems associated with the application of optimal control to greenhouse climate control, such as fast varying unpredictable weather variations and the stiff nature of the system, will be discussed.

## 1. Introduction

The profit obtained from tomato crop production in greenhouses is mainly determined by the production rate, the price of tomatoes, and the costs associated with maintaining a favorable climate in the greenhouse, along with the growers ability to prevent or stop pest or disease development. Present climate control of greenhouses is mainly achieved by trying to maintain predefined setpoints, without considering system constraints and the associated costs of this operation. Furthermore the control of temperature, humidity and carbon dioxide concentration, the main climate variables in the greenhouse, result in conflicting interests concerning heating, ventilation and carbon dioxide supply, the main control variables. At present hierarchical rules guide the choice of setpoints for temperature, carbon dioxide concentration and ventilation.

Given the conflicting interests and the non-linear multivariable nature of the greenhouse-

tomato crop system, together with the desire to consider the costs of maintaining a certain favorable greenhouse climate, the application of optimal control seems natural. Dynamic models of the multivariable greenhouse-tomato crop system are available (Bot 1983, Jones et. al. 1989) even if not thoroughly describing it, mainly because some interactions (pest development versus climate for example) are insufficiently known and may be of a stochastic nature. Given such a model of the multivariable greenhouse-tomato crop system, which describes (part) of the interactions, an integral costfunctional, which reflects the profit (i.e. the income minus the costs) of the tomato crop production, is maximized. In this way we make more intensive use of the knowledge about the greenhouse-tomato crop system while attaining what is truly our goal, namely maximizing profit. Optimal control has recently been applied with success to optimize tomato and lettuce crop growth in greenhouses but on a theoretical basis (Van Henten and Bontsema 1991, Seginer 1991).

Several problems associated with the application of optimal control to the greenhouse-tomato crop system have to be mentioned as well. The system is characterized by both fast and slow dynamics, the first are associated with the greenhouse climate the second with crop growth. The computation of optimal controls for such systems raises numerical difficulties (Kalman 1964). Freedman and Kaplan (1976) presented a way to overcome these problems for systems without external inputs. The greenhouse-tomato crop system behavior however, is heavily influenced by fast changing outside weather conditions, which trigger the fast system dynamics permanently, making it impossible to apply the result of Freedman and Kaplan. The weather presents yet another difficulty. To compute optimal controls the weather should be completely known over the time interval over which the optimization takes place. Since long term weather predictions are unreliable we are confronted with a trade off concerning the choice of this time interval. If for instance we take it small the weather predictions used will be good and we have no numerical difficulties, however we do not consider the system behavior in the long run in this case.

Our first aim is to investigate what can possibly be gained through the application of optimal control, compared to the existing setpoint control strategies. In a first attempt we will focus on temperature and carbon dioxide management, thus ignoring the water balance of

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the greenhouse and the humidity control although it does influence the control. Given the problems mentioned above we will consider the optimization over a short period of time (compared to the growing period) and use artificially generated and measured weather data which we assume to be known in advance (perfect weather prediction is assumed). Over a short period of time the slow dynamics of the system may be regarded constant which simplifies the dynamic model. In section 2 we present the simplified model describing the growth of tomatoes in a greenhouse. In section 3 we present the optimal control problem and numerical solutions, given several types of weather. Finally we conclude the paper in section 4.

## 2. Dynamic model of the greenhouse-tomato crop system.

The model of the greenhouse-tomato crop system is taken from Tchamitchian et. al. (1992) which is mainly based on three models proposed in the literature. Two of them (Bot 1983, Udink ten Cate 1983) describe the greenhouse climate dynamics to which the carbon dioxide balance was added, and the third (Amthor and McCree 1990) describes the dynamics involved in tomato crop growth. Finally the physiological functions which involve photosynthesis and respiration have been taken from Jones et. al. (1989). From the control point of view it is important to distinguish state variables which are typed capital bold, control variables which are typed small bold, external inputs which are capital, dependent quantities which are capital Greek, and constants (system parameters or quantities which are assumed to be constant) which are small.

### 2.1 The greenhouse climate model.

The greenhouse model has to fulfill different requirements:

- it must provide the greenhouse weather variables necessary to the crop growth model, namely the greenhouse air temperature and carbon dioxide content and the greenhouse PAR irradiance (the visible part of the solar radiation);
- forecasts of the external weather (the disturbances) must be available; these are, in general, the minimum and maximum air temperature, wind speed and solar radiation.

Therefore, we choose a simplified greenhouse model (Udink ten Cate 1983) which has two state variables, the greenhouse air and soil temperature,  $T_g$  and  $T_s$ , respectively, and take into account the heating  $h(t)$  and ventilation of the system,

$$dT_g/dt = 1/c_{gr} \{ (\Lambda(t) + k_r)(T_0(t) - T_g(t)) + h(t) + k_s(T_s(t) - T_g(t)) + \eta G(t) \}, \quad (1a)$$

$$dT_s/dt = 1/c_s \{ k_s(T_g(t) - T_s(t)) + k_d(t_d(t) - T_s(t)) \}, \quad (1b)$$

where  $c_s$  and  $c_{gr}$  are the soil and greenhouse heat capacity,  $T_0$  is the outside temperature and

$t_d$  the soil deepest layer temperature and  $G$  the incoming solar radiation while  $\eta$  is the radiation efficiency. Furthermore  $k_r$  is the roof heat transfer coefficient and  $k_d$  the soil to soil heat transfer coefficient. They are related to roof and wall convection exchanges and soil conduction and convection exchanges. The ventilation transfer coefficient  $\Lambda$  is given by

$$\Lambda(t) = m_a c_p \phi_v(t), \quad (1c)$$

where  $m_a$  is the air density,  $c_p$  the air specific heat and  $\phi_v$  the ventilation.

This model approximates all convection heat exchanges to first order processes, while the radiation is taken as a direct heating source. Its efficiency coefficient is more of a calibration parameter and has no real physical meaning; it allows for the indirect taking into account of the infrared radiation exchanges between the greenhouse and the sky vault, which are not described here because the associated weather forecast (IR irradiance from the sky) is not available.

A classic carbon dioxide balance has been added to this model, including the crop activity,

$$dC_g/dt = a_g/v_g \{ \phi_v(t)(C_0(t) - C_g(t)) + c_i(t) \cdot \Theta_c(t) - \Omega_c(t) \}, \quad (1d)$$

where  $C_g$  is the carbon dioxide concentration in the greenhouse  $a_g$  and  $v_g$  are the greenhouse area and volume respectively,  $C_0$  is the outside carbon dioxide concentration,  $\Theta_c$  and  $\Omega_c$  the respiration and photosynthesis of the tomato crops respectively, while  $c_i$  is the carbon dioxide injection.

The ventilation, controlled by the window opening and driven by the wind speed is calculated by an empirical relationship, calibrated on the same greenhouse as the greenhouse model,

$$\phi_v(t) = a_v + b_v W(t) + c_v W(t) r_w(t), \quad (1e)$$

where  $a_v$ ,  $b_v$  and  $c_v$  are renewal rate parameters,  $W$  is the wind speed and  $r_w$  the relative window aperture.

Finally, the greenhouse PAR irradiance  $\Gamma_g$ , which is an external input to the crop model described in section 2.2, is calculated by an algebraic equation assuming a constant greenhouse transmission  $t_g$  and a constant ration  $p$  between solar radiation  $G$  and its PAR fraction,

$$\Gamma_g(t) = t_g p G(t). \quad (1f)$$

### 2.2 The crop model.

Basically we are interested in the fruit production of the crop, which in our case is a continuous process because of the structure of the greenhouse tomato crop which bears at the same moment fruits of different ages, ranking from fecundated flowers to ripe and ready to harvest tomatoes.

Because of the long term unpredictable nature of the weather and the problems associated with the stiff nature of the greenhouse tomato crop system we concentrate on the short term optimization of the tomato-greenhouse system and we will use a simplifying hypotheses.

While in the production stage, the tomato can be assimilated to a steady state system in which the apparition of new young fruits is compensated for by the harvest of old ones. Over short time intervals, the change of weight of the leaves and stems is little compared to that of the fruits and can be neglected. Thus we make the following hypotheses,

- the crop dry weight and leave area index are external variables that do not change during the optimization;
- the dry weight accumulated in the fruits and which should be harvested some time later is immediately removed and given the value of the selling tomatoes.

The harvest  $\Delta_g$  in this case is proportional to the difference of photosynthesis and respiration i.e.

$$\Delta_g(t) = g_c(\Omega_c(t) - \Theta_c(t)) \quad (2a)$$

where  $g_c$  is the growth conversion efficiency, a factor taking into account the respiratory cost of the growth.

Classical relationships have been used to describe the photosynthesis  $\Omega_c$  and the respiration  $\Theta_c$ . (Jones et. al. 1989). For the photosynthesis we have,

$$\Omega_c(t) = \Sigma(t) / k \log \left[ \frac{(1-m)\Sigma(t) \cdot ck\Gamma_q(t)}{(1-m)\Sigma(t) + ck\Gamma_q(t) \cdot e^{-kl}} \right], \quad (2b)$$

where  $c$  is the leaf quantum yield efficiency,  $k$  the light extinction coefficient,  $m$  the leaf light transmission factor, and  $l$  the leaf area index. For  $\Sigma(t)$  we have,

$$\Sigma(t) = \tau_c C_q(t) t_{eff}^M \left[ 1 - \frac{(T_q(t) - t_{min})(T_q(t) - t_{max})}{(T_q(t) - t_{min})^2 + (T_q(t) - t_{max})^2 + t_c} \right], \quad (2c)$$

where  $t_{min}$ ,  $t_{max}$ ,  $t_c$  and  $t_{eff}$  are known temperatures and  $\tau_c$  is the leaf carbon dioxide efficiency. For the respiration we have,

$$\Theta_c = m_r q \frac{T_q(t) - 20}{10} \rho v \quad (2d)$$

where  $m_r$  is the ratio between  $CO_2$  and  $CH_2O$  molar masses,  $q$  the respiration temperature factor,  $\rho$  the maintenance respiration factor and  $v$  the total dry weight of the tomato crops.

We are aware that the dynamics of the crop growth process is ignored in this very simple model and especially the buffering facility provided by the long period of dry matter

accumulation in the fruits. But there seem to be no simple way to include them when considering the crop on the basis of a one or two days periods. By ignoring this dynamics, we introduce the following pattern in the model: during night period, when there is no light, no gain is possible but losses of dry matter (and thus harvest) occur due to the respiration; harvest is only accumulated during daylight period.

### 3. Optimal control problem and numerical solutions.

Given a dynamic model of the controlled system in state space form i.e.,

$$dx/dt = f(x(t), u(t), d(t), t), \quad (3)$$

where the vector  $x$  contains the state variables, the vector  $u$  the control variables, and the vector  $d$  the external inputs an optimal control problem is to find the control  $u(t)$  which minimizes the integral costfunctional,

$$J(u(t), x(t_0)) = \int_{t_0}^{t_f} L(x(t), u(t), d(t), t) dt \quad (4)$$

where  $t_0$  and  $t_f$  determine the interval over which the integral is minimized while we assume  $x(t_0)$  to be known (Lewis 1986). The quantity  $L$  is referred to as the cost function. The dynamics (1), (2) can be written in the form (3). The profit over some time interval equals the income from the harvested tomatoes, assuming a fixed price over the growing period, minus the costs of maintaining the greenhouse climate during that time interval which for instance involves the price of carbon dioxide gas and the price of the energy needed to heat the greenhouse. Since (4) is minimized to maximize profit we obtain,

$$L(x, u, d, t) = -[\alpha_1 \Delta_g(t) - \alpha_2 c_1(t) - \alpha_3 h(t)], \quad (5)$$

where  $\alpha_1$  is the price of tomatoes, expressed in dry matter to match the units of the harvest, and  $\alpha_2$  and  $\alpha_3$  are the costs of carbon dioxide and the heating. The window operation does not involve any cost. For detailed information concerning the model and costfunctional we refer to Tchamitchian et. al. (1991).

The model and the costfunctional turn out to be linear in the controls. From optimal control theory it is well known that in this case the optimal control is of bang-bang type if the problem is non-singular. The optimal control may be obtained by solving an associated two-point-boundary-value problem (Kalman 1964, Lewis 1986). However in most cases of bang-bang control the associated two-point-boundary-value problem (TPBVP) is very sensitive to boundary conditions and integration errors causing numerical difficulties. For these problems first order gradient algorithms (Kalman 1964), designed to solve optimal control problems, are much more appropriate since they suffer much less from sensitivity problems. We have chosen a "first order strong variations algorithm" described by Mayne and Polak (1975) which differs from first order gradient algorithms only in the sense that the adjustment of the controls at each iteration is performed in a somewhat different manner which may result in

faster convergence. We computed several optimal control histories in case of both artificial smooth and measured weather data. In the latter case the optimal control exhibits several switches because of the fast weather variations. Figure 1 illustrates the main results obtained with the artificial weather. The maximum profit obtained is 0.062 Dutch guilders per square meter over this day. Figure 2 amounts to weather recorded at 2 October 1991. The maximum profit in this case equaled 0.098 Dutch guilders per square meter on this day.

The results obtained with the artificial weather data set exhibit little use of the controls, maintaining the windows open at night and increasing the carbon dioxide content of the greenhouse in the first part of the day. A short pulse of enrichment occurs by the end of the day to increase again the carbon dioxide content, just after the windows were opened again, likely because the wind is now low enough to limit the losses towards the outside air. The underlying strategy is to reduce the dry matter losses at night by lowering the temperature (using the outside air to cool the greenhouse) and then increasing the photosynthesis by carbon dioxide enrichment. A trade off is found between the costs of carbon dioxide leakage (enrichment and windows opened) and the associated gain. No use of the heating is made, because the only requirement occurs during daylight period (photosynthesis need a minimum 14°C) and then the solar radiation provides enough energy to satisfy them.

When using real weather data sets, the results show a much more intensive use of the control but still pertain the same strategy. Window control at night mainly depends on the variations of the outside air temperature. When it decreases, they are opened for cooling purposes, and closed when it increases to keep the fresh air inside the greenhouse. This is completely true in the final part of the integration period where no interaction with the carbon dioxide control can happen (it is not worth to use it because there will be no photosynthesis to consume profitably this carbon dioxide). Again heating does not occur.

These results do not fit the common practice which is to maintain the temperature above a minimum 15 to 16°C and enrich to a maximum level of 1000 ppm carbon dioxide concentration. The physiological basis of these practices are not all clearly stated; however, some improvements of the model are readily feasible, among which a better description of the carbon dioxide effect on the photosynthesis.

#### 4. Conclusions

Although we have been able to compute optimal greenhouse climate controls it is still difficult to compare the results to other control strategies. This is mainly due to the following facts,

- a) Heating does not occur since the temperature requirements of the plant are badly described by the model which causes heating to have no associated income.
- b) Humidity control was excluded from the optimization but turns out to fundamentally influence the control.

Furthermore we assumed the weather to be completely known in advance, which is unrealistic. Finally the results of course depend on the choice of the initial time, the corresponding initial conditions, and the final time of the optimization. As explained this choice is by no means obvious.

A number of things remain to be investigated. A fundamental question remains how to actually apply optimal control in practice given the problems caused by the long term unpredictable and fast varying weather conditions in combination with the fast and slow system dynamics. Simulation experiments may help to answer this. Obviously humidity control should be included in the future. The dynamics used to describe the greenhouse-tomato crop system contained several simplifications a number of which are questionable and need further investigation. The optimal solutions have to be confronted with other, possibly non-deterministic, knowledge about the system (e.g. grower's knowledge and experience) to further improve the model and the costfunctional.

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Fig 1a: Window control

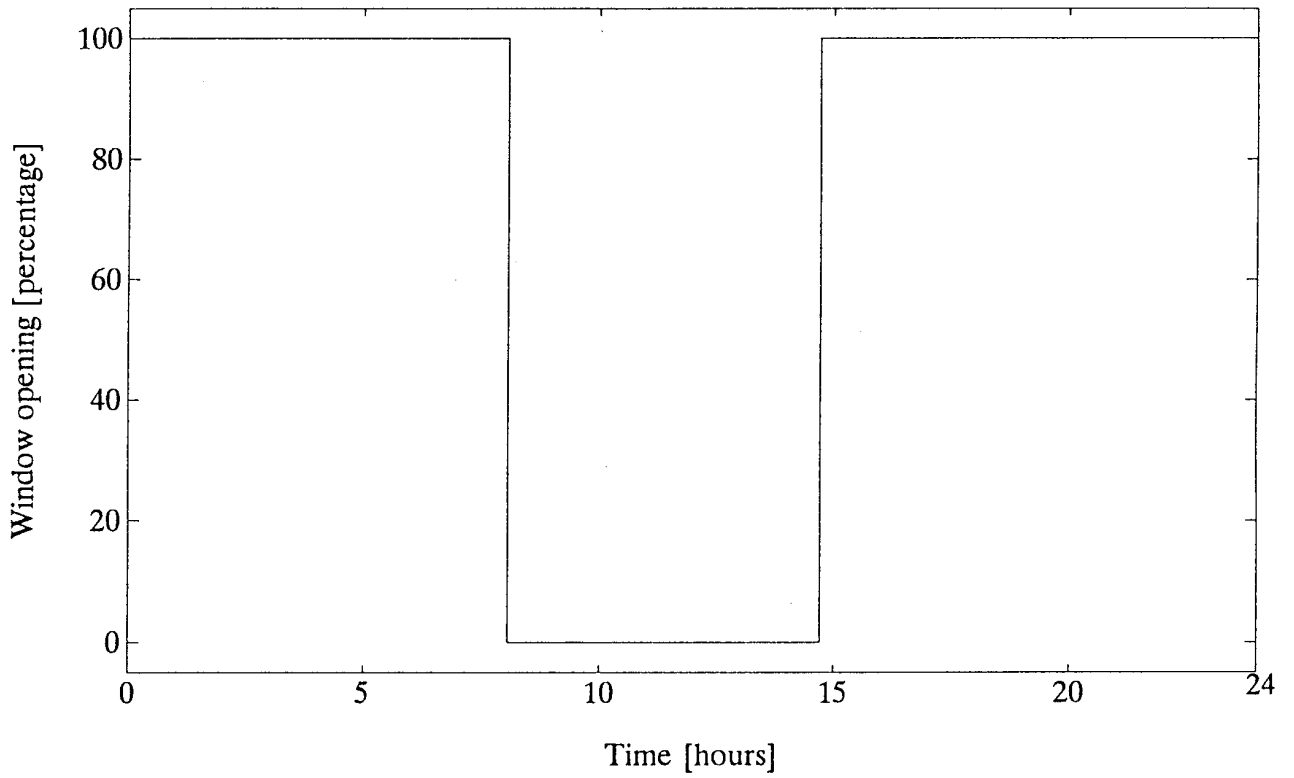


Fig 1b: Carbon dioxide control

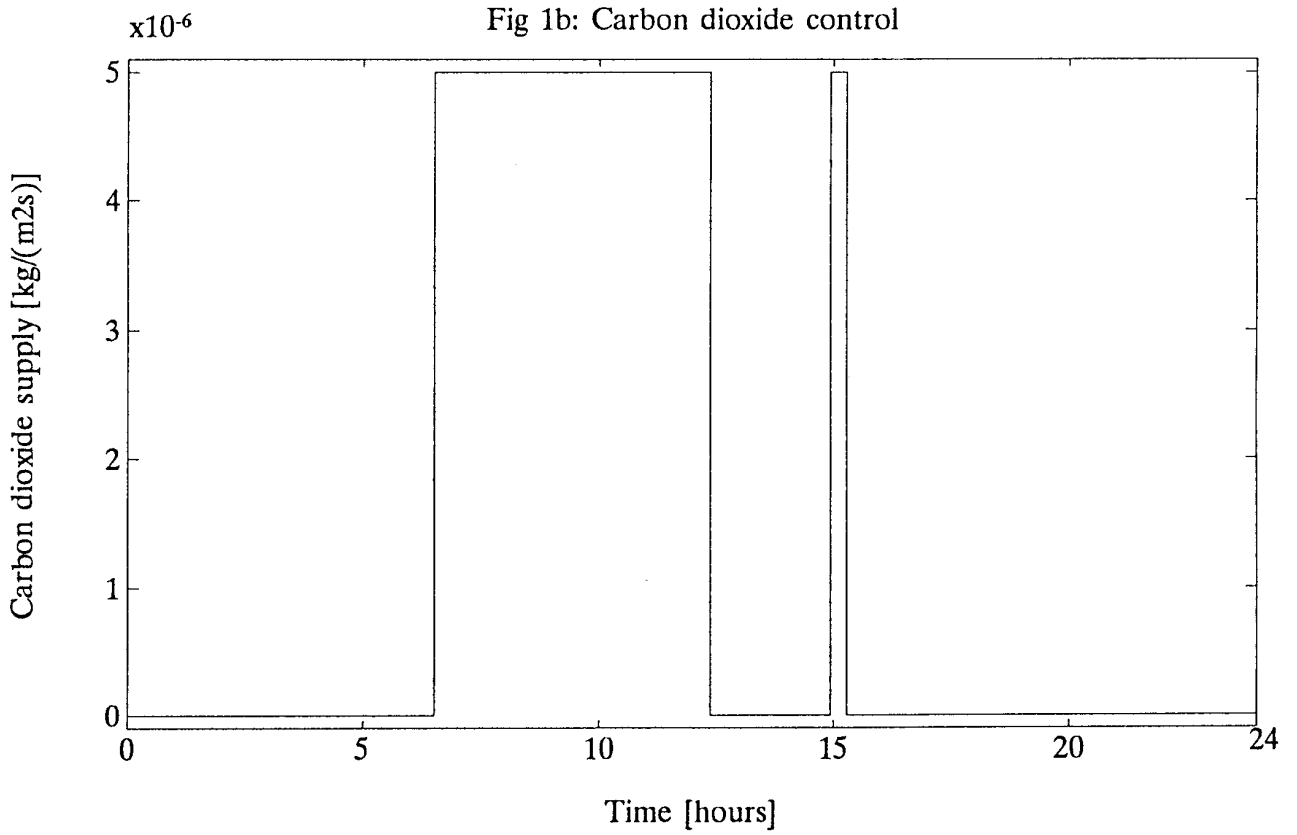


Fig 1c: State variables

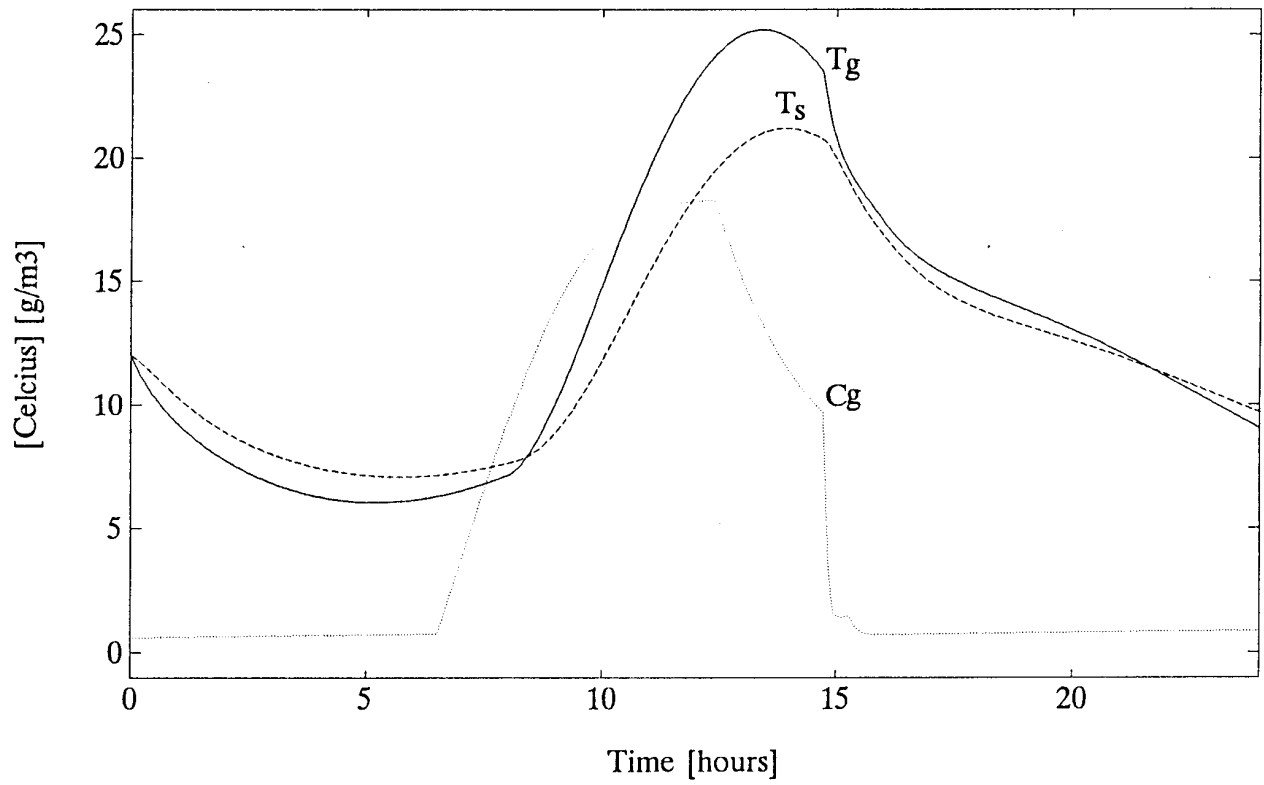


Fig 1d: External inputs (weather)

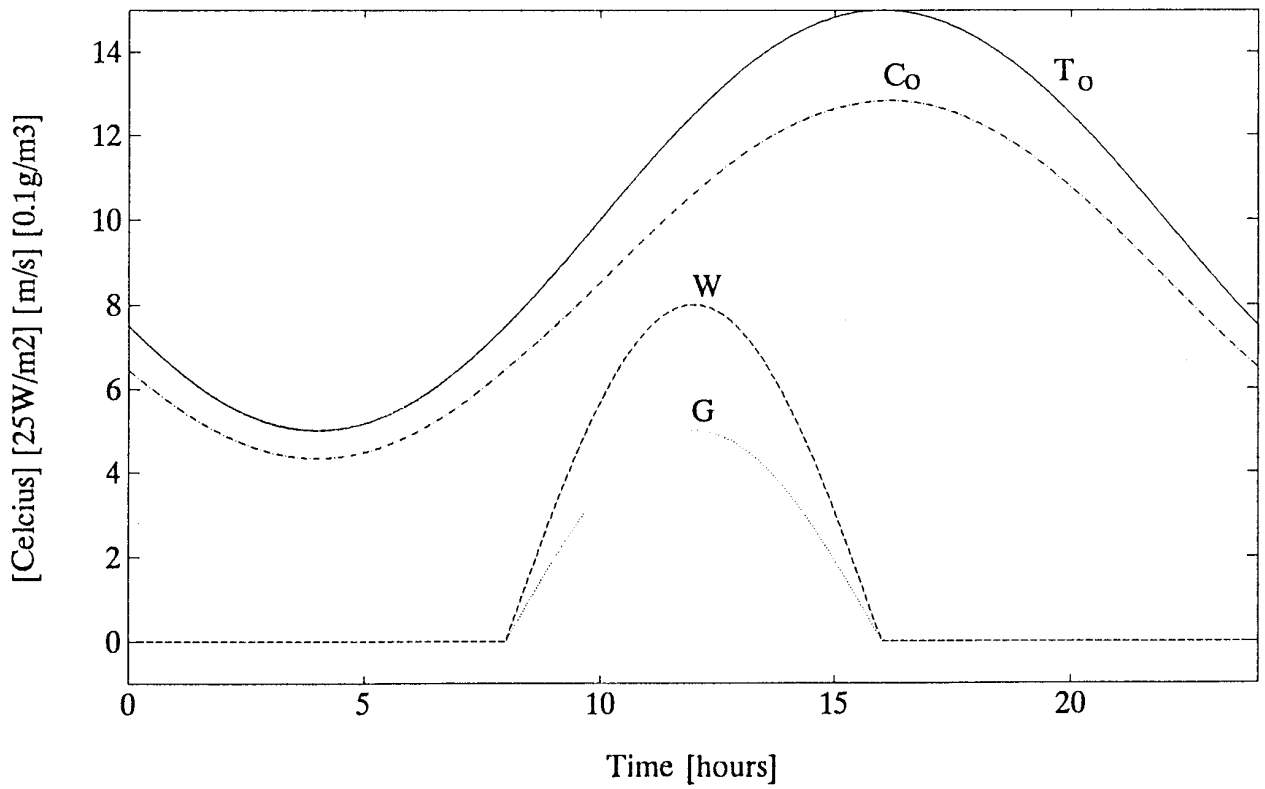


Fig 2a: Window control

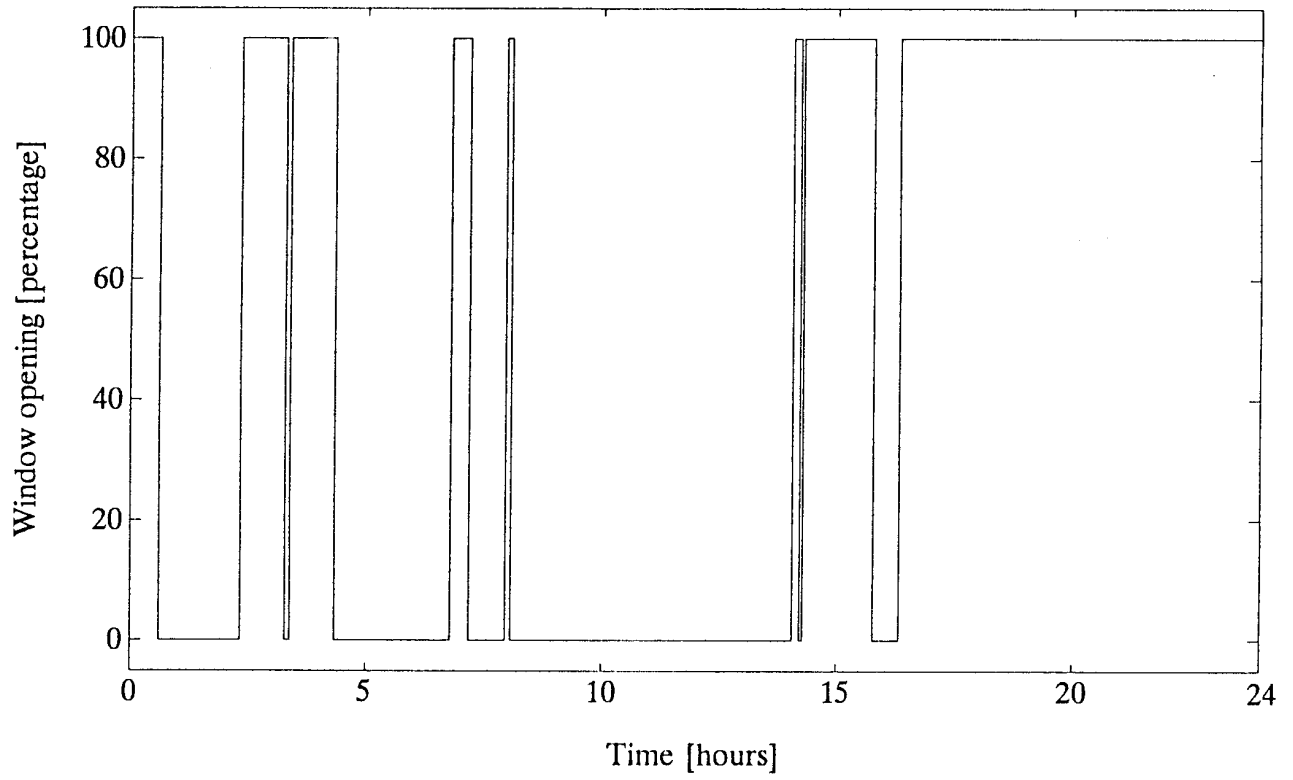


Fig 2b: Carbon dioxide control

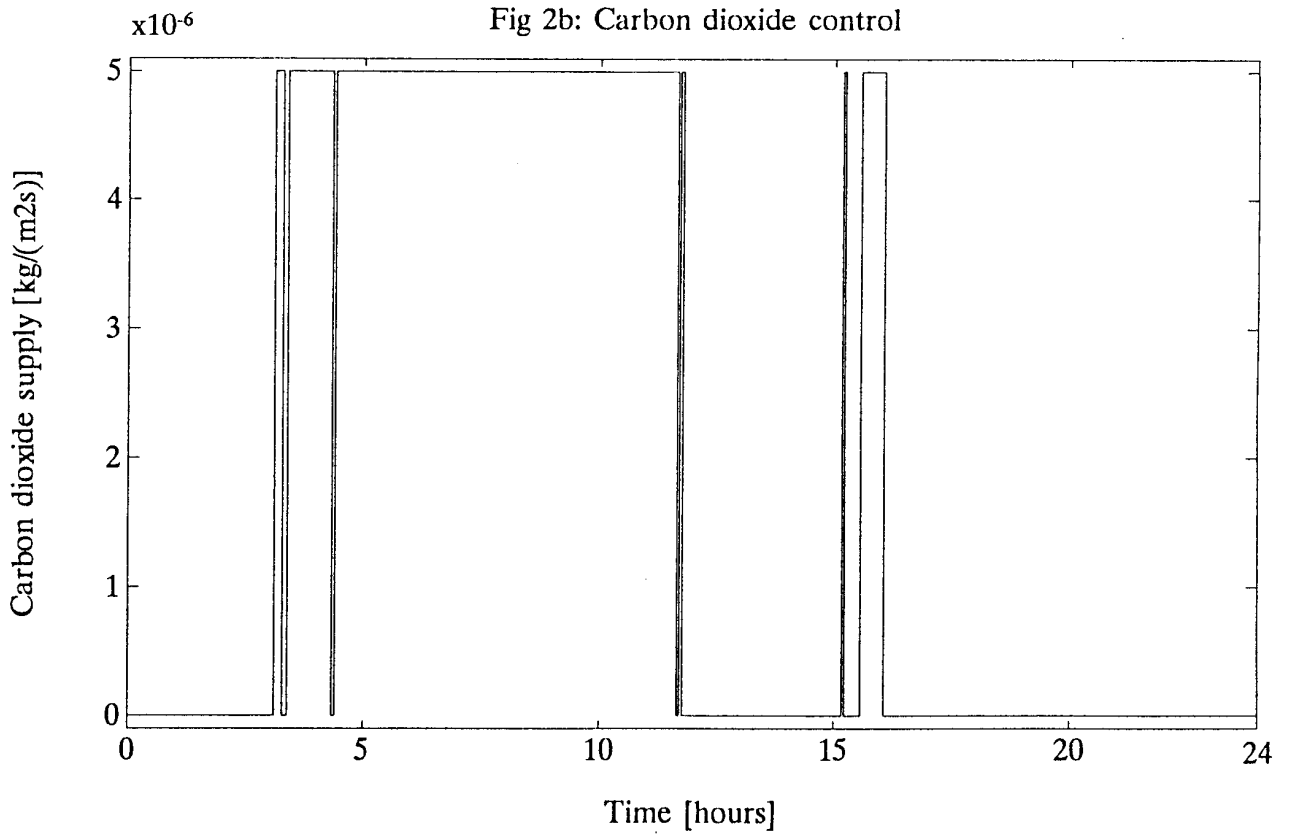


Fig 2c: State variables

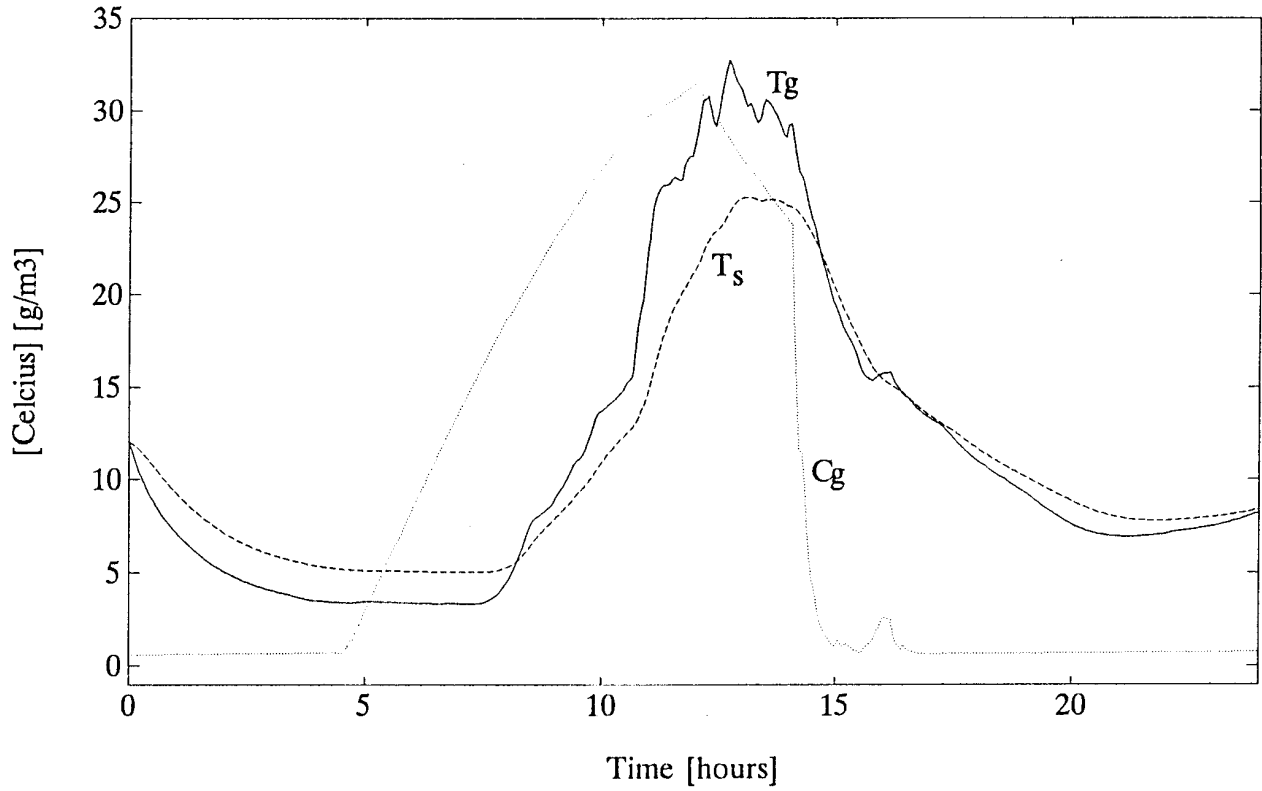


Fig 2d: External inputs (weather)

