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Brief Paper

Minimal and non-minimal optimal fixed-order compensators for time-varying discrete-time systems☆

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Abstract

Using the minimality property of finite-horizon time-varying compensators, established in this paper, and the Moore–Penrose pseudo-inverse instead of the standard inverse, strengthened discrete-time optimal projection equations (SDOPE) and associated boundary conditions are derived for finite-horizon fixed-order LQG compensation. They constitute a two-point boundary value problem explicit in the LQG problem parameters which is equivalent to first-order necessary optimality conditions and which is suitable for numerical solution. The minimality property implies that minimal compensators have time-varying dimensions and that the finite-horizon optimal full-order compensator is not minimal. The use of the Moore–Penrose pseudo-inverse is further exploited to reveal that the optimal projection approach can be generalised, but only to partially include non-minimal compensators. Furthermore, the structure of the space of optimal compensators with arbitrary dimensions is revealed to a large extent. Max–min compensator dimensions are introduced and their significance in solving numerically the two-point boundary value problem is explained. The numerical solution is presented in a recently published companion paper, which relies on the results of this paper.

Keywords: Optimal; Reduced-order; LQG; Control

1. Introduction

It is well known that the optimal full-order LQG compensator realises the best performance obtainable with any compensator, regardless of its dimensions (Kwakernaak & Sivan, 1972). Therefore, in practice a priori fixed compensator dimensions will always be chosen less than those of the system. Then optimal fixed-order compensator design becomes optimal reduced-order compensator design. Given the practical objective to reduce the dimensions of the compensator, minimal compensators are the interesting ones. In addition, in the time-invariant infinite-horizon case the minimality of the compensator enables the use of the standard inverse in deriving the optimal projection equations (Hyland & Bernstein, 1984). As demonstrated in this paper, the use of the Moore–Penrose pseudo-inverse, which is needed in the time-varying case, significantly complicates the derivation.

Given the restriction to minimal compensators, the practical problem with the pre-specification of compensator dimensions is that the optimal compensator with these pre-specified dimensions may not be minimal (Yousuff & Skelton, 1984; Van Willigenburg & De Koning, 2000). Therefore, an important issue is the selection of prescribed compensator dimensions which guarantee a priori the minimality of the optimal compensator. This issue is resolved in this paper using the notion of max–min compensator dimensions, established in this paper. This constitutes one of the main practical result of this paper.

Except for the companion paper, Haddad and Tadmor (1993) seem to be the only one who applied the optimal projection approach to a *finite-horizon* fixed-order problem. In this case, in addition to the optimal projection equations, boundary conditions determine the first-order

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necessary optimality conditions, which now constitute a two-point boundary value problem. Haddad and Tadmor (1993), who considered the continuous time-varying case, could not specify the boundary conditions in terms of the parameters making up the problem. This prevents the development of numerical algorithms. In this paper, this problem is explained and resolved by introducing a finite-horizon minimality property and by using the Moore–Penrose pseudo-inverse. This constitutes another main practical result of this paper. The use of the Moore–Penrose pseudo-inverse is further exploited to largely reveal the structure of the space of optimal compensators. This is the main theoretical result of the paper.

For the problem formulation and several other important results we refer to the companion paper (Van Willigenburg & De Koning, 1999a). Although the results of this paper also apply to systems with white parameters, considered in the companion paper, to simplify the notation we will consider systems with deterministic parameters here. In that case, all terms in the companion paper involving a tilde should be deleted and also all the overbars appearing above matrix expressions. The cross covariance matrix of the system and the cross term in the criterion are also deleted. Then the notation complies with the one used in this paper.

2. Minimality of finite-horizon time-varying discrete-time compensators

Consider the following deterministic time-varying compensator defined over a finite horizon:

$$\hat{x}_{i+1} = F_i\hat{x}_i + K_i y_i, \quad \hat{x}_i \in R^{n_i^c},$$
$$y_i \in R^{l_i}, \; i = 0,1,\dots,N-1. \tag{1}$$

Denote this compensator by $(\hat{x}_0, F^N, K^N)$ where $F^N = \{F_0, F_1, \dots, F_{N-1}\}$ and where $K^N = \{K_0, K_1, \dots, K_{N-1}\}$.

For compensator (1) we have

$$\hat{x}_i = F_{i,0}\hat{x}_0 + \sum_{k=0}^{i-1} F_{i,k+1}K_k y_k, \quad i = 1,\dots,N, \tag{2}$$

where

$$F_{l,m} = F_{l-1}F_{l-2}\dots F_m \; l > m, \quad F_{l,m} = I_{n_l^c} \; l = m. \tag{3}$$

Definition 1. $(0, F^N, K^N)$ is called reachable if for $\forall \hat{x} \in R^{n_i^c}$, $\forall\, i = 1,2,\dots,N$, $\exists\{y_0,\dots,y_{i-1}\}$ such that $\hat{x}_i = \hat{x}$ can be reached.

Consider the following deterministic time-varying compensator defined over a finite horizon:

$$\hat{x}_{i+1} = F_i\hat{x}_i, \quad \hat{x}_i \in R^{n_i^c},$$
$$u_i = L_i\hat{x}_i, \quad u_i \in R^{m_i}, \quad i = 0,1,\dots,N-1. \tag{4}$$

Denote this compensator by (F^N, L^N), where $L^N = \{L_0, L_1, \dots, L_{N-1}\}$.

Definition 2. (F^N, L^N) is called observable if for $\forall\, i = 0,1,\dots,N-1$, $u_i = 0$, $u_{i+1} = 0,\dots,u_{N-1} = 0$ implies $\hat{x}_i = 0$.

Consider the compensator

$$\hat{x}_{i+1} = F_i\hat{x}_i + K_i y_i, \quad \hat{x}_i \in R^{n_i^c}, \; y_i \in R^{l_i},$$
$$u_i = L_i\hat{x}_i, \quad u_i \in R^{m_i}, \; i = 0,1,\dots,N-1. \tag{5}$$

Denote this compensator by $(\hat{x}_0, F^N, K^N, L^N)$.

A non-zero initial condition $\hat{x}_0$ and the boundary condition $\hat{x}_N$ complicate the definition of a minimality property over a finite horizon. From Eq. (5) observe that $\hat{x}_N$ does not influence the input–output behaviour of the compensator $(\hat{x}_0, F^N, K^N, L^N)$, so its minimal dimension $n_N^c = 0$. Since $\hat{x}_0$ is deterministic, at time $i = 0$ a basis transformation exists such that at most one compensator state variable of the transformed $\hat{x}_0$ is unequal to zero. Therefore, $n_0^c = 1$ is the minimal dimension of $\hat{x}_0$ that preserves the input–output behaviour.

Definition 3. $(0, F^N, K^N, L^N)$ is called minimal if $(0, F^N, K^N)$ is reachable and (F^N, L^N) is observable and if in addition $n_0^c = 1$ and $n_N^c = 0$.

The following analysis explains why Definition 3 must be *generalised* for compensators with $\hat{x}_0 \neq 0$. Consider the sets $\{\hat{x}_i^r \mid \hat{x}_i = \hat{x}_i^r\}$ of states that can be reached by the compensator $(\hat{x}_0, F^N, K^N)$ at each time $i = 1,2,\dots,N$ using $y_0, y_1, \dots, y_{i-1}$. These sets are determined by Eq. (2). The first term on the right in Eq. (2) is a constant term while the second term, through the variation of $y_0, y_1, \dots, y_{i-1}$, either represents the full compensator state-space at time i, i.e. $R^{n_i^c}$, or it represents a hyperplane with dimension $n_i^{cr} < n_i^c$ inside the state-space $R^{n_i^c}$. In the latter case, since the hyperplane represented by the second term contains the origin, a basis transformation exists such that n_i^{cr} unit vectors of the new basis span this hyperplane. If the first term is part of this hyperplane, which is always the case if $\hat{x}_0 = 0$, then it does not change the hyperplane. If not, the first term shifts the hyperplane away from the origin. Then, to represent the hyperplane, *one additional unit vector*, i.e. $n_i^{cr} + 1$ state variables, is needed.

Let $W_{0,i} \in R^{n_i^c \times n_i^c}$ denote the reachability grammian of the compensator $(0, F^N, K^N)$ associated to the state transition $\hat{x}_0 = 0$ to $\hat{x}_i = \hat{x}$, $i \in [1, N]$, i.e.,

$$W_{0,i} = \sum_{k=0}^{i-1} F_{i,k+1}K_k K_k^T F_{i,k+1}^T, \quad i = 1,2,\dots,N. \tag{6}$$

Based on Eq. (2) define

$$W'_{0,i} = F_{i,0}\hat{x}_0\hat{x}_0^T F_{i,0}^T + \sum_{k=0}^{i-1} F_{i,k+1}K_kK_k^T F_{i,k+1}^T,$$

$$i = 1,2,\ldots,N. \tag{7}$$

Eq. (7) may be interpreted as a grammian associated to the compensator state transition from $\hat{x}_0$ to $\hat{x}_i = \hat{x}$, $i\in[1,N]$. Dual to the reachability grammian (6) consider the observability grammian $M_i^N \in R^{n_i^c \times n_i^c}$ given by

$$M_{i,N} = \sum_{k=i}^{N-1} F_{k,i}^T L_k^T L_k F_{k,i}, \quad i = 0,1,\ldots,N-1. \tag{8}$$

Then from Definitions 1–3 and Eqs. (2), (6)–(8) the following two lemmas are immediate.

Lemma 1. $(0,F^N,K^N)$ *reachable* $\Leftrightarrow W_{0,i}$ *full rank* $\forall\ i\in[1,N]$. *Dually* (F^N,L^N) observable $\Leftrightarrow M_{i,N}$ *full rank* $\forall\ i\in[0,N-1]$

Lemma 2. (1) *The first term on the right in Eq.* (2) *lies inside the hyperplane with dimension* $n_i^{cr} < n_i^c$ *determined by the second term on the right in Eq.* (2) $\Rightarrow rank(W'_{0,i}) = n_i^{cr} < n_i^c$, $i\in[1,N]$.

(2) *The first term on the right in Eq.* (2) *lies outside the hyperplane with dimension* $n_i^{cr} < n_i^c$ *determined by the second term on the right in Eq.* (2) $\Rightarrow rank(W'_{0,i}) = n_i^{cr} + 1 \leqslant n_i^c$, $i\in[1,N]$.

(3) *The second term on the right in Eq.* (2) *spans the full state-space* $R^{n_i^c} \Rightarrow rank(W'_{0,i}) = n_i^c$, $i\in[1,N]$.

From Lemma 2 and the analysis following Definition 3 $rank(W'_{0,i})$ represents precisely the minimum number of compensator state variables needed to describe the reachable space at time $i\in[1,N]$.

Definition 4. $(\hat{x}_0,F^N,K^N,L^N)$ is called minimal if $\forall i\in[0,N-1]$, $M_{i,N}$ full rank and if $\forall i\in[1,N], W'_{0,i}$ full rank and if in addition $n_0^c = 1$ and $n_N^c = 0$.

It is well known that the reachability and observability grammian (6), (8) can be given in recursive form as follows:

$$W_{0,i+1} = F_iW_{0,i}F_i^T + K_iK_i^T,$$

$$i = 0,1,\ldots,N-1,\ W_{0,0} = 0\in R^{n_0^c\times n_0^c}, \tag{9}$$

$$M_{i,N} = F_i^TM_{i+1,N}F_i + L_i^TL_i, \quad i = 0,1,\ldots,N-1,$$

$$M_{N,N} = 0\in R^{n_N^c\times n_N^c}. \tag{10}$$

Similar to (9) the recursive form of (7) is given by

$$W'_{0,i+1} = F_iW'_{0,i}F_i^T + K_iK_i^T, \quad i = 0,1,\ldots,N-1,$$

$$W'_{0,0} = \hat{x}_0\hat{x}_0^T \in R^{n_0^c\times n_0^c}. \tag{11}$$

Eqs. (9) and (11) are identical except for the initial value 0 which is changed into $\hat{x}_0\hat{x}_0^T$. This constitutes the generalisation. Introduce

$$r_i^c = \min(rank(W'_{0,i}), rank(M_{i,N})), \quad i = 0,1,\ldots,N \tag{12}$$

Then from Eqs. (10)–(12)

$$\hat{x}_0 \neq 0 \Rightarrow r_0^c = 1, \quad \hat{x}_0 = 0 \Rightarrow r_0^c = 0, \quad r_N^c = 0, \tag{13a}$$

$M_{0,N} \neq 0$

$$r_i^c - m_i \leqslant r_{i+1}^c \leqslant r_i^c + l_i, \quad i\in[0,N-1]. \tag{13b}$$

From Eq. (13) and Definition 4 the dimensions of a minimal compensator satisfy

$$n_i^c = r_i^c, \quad i = 1,2,\ldots,N-1, \quad n_0^c = 1, \quad n_N^c = 0. \tag{14}$$

On the other hand if $(\hat{x}_0,F^N,K^N,L^N)$ has dimensions n_i^c satisfying (14) then one can always choose the compensator such that it is minimal. Eqs. (14) and (13b) imply that the *change* of the dimension of the state of a minimal compensator, from one discrete-time instant to the next, is bounded from above and below.

3. First-order necessary optimality conditions

Similar to Haddad and Tadmor (1993), Van Willigenburg and De Koning (2000) first-order necessary optimality conditions for the solution of the optimal fixed-order compensation problem can be presented in the form of a two-point boundary value problem in which P_i^1, P_i^{12}, P_i^2, S_i^1, S_i^{12}, S_i^2, $i = 0,1,\ldots,N$ have to be solved. These two triples of matrices are, respectively the partitioning of the symmetric second moment matrix of the closed-loop system and the dual closed-loop system. The stationarity conditions, which must hold at each discrete-time instant $i = 0,1,\ldots,N$ are,

$$S_{i+1}^{12^T}\Phi_iP_i^{12} - S_{i+1}^{12^T}\Gamma_iL_iP_i^2 + S_{i+1}^2F_iP_i^2$$

$$+ S_{i+1}^2K_iC_iP_i^{12} = 0, \tag{15a}$$

$$S_{i+1}^{12^T}\Phi_iP_i^1C_i^T - S_{i+1}^{12^T}\Gamma_iL_iP_i^{12^T}C_i^T + S_{i+1}^2F_iP_i^{12^T}C_i^T$$

$$+ S_{i+1}^2K_iC_iP_i^1C_i^T + S_{i+1}^2K_iW_i = 0, \tag{15b}$$

$$-\Gamma_i^TS_{i+1}^1\Phi_iP_i^{12} - \Gamma_i^TS_{i+1}^{12}K_iC_iP_i^{12} - \Gamma_i^TS_{i+1}^{12}F_iP_i^2$$

$$+ \Gamma_i^TS_{i+1}^1\Gamma_iL_iP_i^2 + R_iL_iP_i^2 = 0, \tag{15c}$$

$$S_0^{12^T}\hat{x}_0 + S_0^2\hat{x}_0 = 0. \tag{15d}$$

The six dynamic equations with boundary conditions at the initial and final time are

$$\Phi_i^T S_{i+1}^1 \Phi_i + C_i^T K_i^T S^{12T}_{i+1} \Phi_i + \Phi_i^T S_{i+1}^{12} K_i C_i + C_i^T K_i^T S_{i+1}^2 K_i C_i + Q_i = S_i^1, \quad S_N^1 = Z, \tag{15e}$$

$$\Phi_i^T S_{i+1}^{12} F_i + C_i^T K_i^T S_{i+1}^2 F_i - \Phi_i^T S_{i+1}^1 \Gamma_i L_i - C_i^T K_i^T S_{i+1}^{12T} \Gamma_i L_i = S_i^{12}, \quad S_N^{12} = 0, \tag{15f}$$

$$L_i^T \Gamma_i^T S_{i+1}^1 \Gamma_i L_i - F_i^T S_{i+1}^{12^T} \Gamma_i L_i - L_i^T \Gamma_i^T S_{i+1}^{12} F_i + F_i^T S_{i+1}^2 F_i + L_i^T R_i L_i = S_i^2, \quad S_N^2 = 0, \tag{15g}$$

$$\Phi_i P_i^1 \Phi_i^T - \Gamma_i L_i P_i^{12T} \Phi_i^T - \Phi_i P_i^{12} L_i^T \Gamma_i^T + \Gamma_i L_i P_i^2 L_i^T \Gamma_i^T + V_i = P_{i+1}^1, \quad P_0^1 = \overline{x_0 x_0^T}, \tag{15h}$$

$$\Phi_i P_i^1 C_i^T K_i^T - \Gamma_i L_i P_i^{12T} C_i^T K_i^T + \Phi_i P_i^{12} F_i^T - \Gamma_i L_i P_i^2 F_i^T = P_{i+1}^{12}, \quad P_0^{12} = \bar{x}_0 \hat{x}_0^T, \tag{15i}$$

$$K_i C_i P_i^1 C_i^T K_i^T + F_i P_i^{12^T} C_i^T K_i^T + K_i C_i P_i^{12} F_i^T + F_i^T P_i^2 F_i + K_i W_i K_i^T = P_{i+1}^2, \quad P_0^2 = \hat{x}_0 \hat{x}_0^T. \tag{15j}$$

The two-point boundary value problem (15) is stated explicitly in terms of the problem parameters. Apart from the time-varying nature and the dependence on boundary conditions (15) strongly resembles the optimality conditions in the infinite-horizon time-invariant case (Van Willigenburg & De Koning, 2000). Therefore, our main theorem is obtained through the generalisation of results presented by Haddad and Tadmor (1993) and Van Willigenburg and De Koning (2000). Since the proof of the main theorem in this section requires the use of the Moore–Penrose pseudo-inverse, instead of the standard inverse, the following generalised lemma is needed.

Lemma 3 (Generalisation of Lemma 1 in Van Willigenburg & De Koning, 1999a). *Suppose $\hat{P},\hat{S} \in R^{n \times n}$ are symmetric non-negative definite and rank$(\hat{P}\hat{S}) = r^c$. Let $G, H \in R^{n^c \times n}$, $n^c \geqslant r^c$ be equal to those in Lemma 1 of the companion paper extended with $n^c - r^c$ rows of zeros. Then from Lemma 1 in the companion paper,*

$$G^T H = \tau = \hat{P}\hat{S}(\hat{P}\hat{S})^{\#}, \tag{16a}$$

$$rank(G) = rank(H) = rank(\tau) = r^c. \tag{16b}$$

The matrix pair G, H and also all matrix pairs AG, AH, where $A \in R^{n^c \times n^c}$ is an arbitrary unitary matrix, are called generalised projective factorisations of $\hat{P}\hat{S}$. They all satisfy equation (16). So up to unitary basis transformations in R^{n^c} all possible projective factorisations in Lemma 1 of the companion paper correspond one to one with all generalised projective factorisation in this lemma.

In the optimal fixed-order compensation problem the dimensions n_i^c, $i = 0,1,\ldots,N$ of the compensator are a priori fixed. However, we may investigate whether a compensator with *arbitrary* dimensions n_i^c, $i = 0,1,\ldots,N$ is a global or local minimum of the *associated* fixed-order compensation problem, i.e. with prescribed compensator dimensions n_i^c, $i = 0,1,\ldots,N$, determined by the compensator under investigation. From here on, unless stated otherwise, this viewpoint will be adopted. Given the LQG problem the recursions (15e)–(15j) can be computed for arbitrary compensators, with arbitrary dimensions n_i^c, $i = 0,1,\ldots,N$, irrespective of whether (15a)–(15d) are satisfied.

Definition 5. Let Λ denote the set of all compensators with arbitrary dimensions n_i^c, $i = 0,1,\ldots,N$. Let Λ_+ denote the set of *pseudo-minimal* compensators with arbitrary dimensions n_i^c, $i = 0,1,\ldots,N$ and with the properties

$$F_i = S_{i+1}^{2\#} S_{i+1}^2 F_i P_i^2 P_i^{2\#}, \quad i = 0,1,\ldots,N-1, \tag{17a}$$

$$K_i = S_{i+1}^{2\#} S_{i+1}^2 K_i, \quad i = 0,1,\ldots,N-1, \tag{17b}$$

$$L_i = L_i P_i^2 P_i^{2\#}, \quad i = 0,1,\ldots,N-1, \tag{17c}$$

$$\hat{x}_0 = S_0^{2\#} S_0^2 \hat{x}_0, \tag{17d}$$

where P_i^2, S_i^2, $i = 0,1,\ldots,N$ are determined by recursions (15e)–(15j). Let $\Lambda_{\min}$ denote the set of *minimal* compensators in the sense of Definition 4.

Definition 6. Let Ω denote the set of all compensators $(\hat{x}_0, F^N, K^N, L^N)$ with arbitrary dimensions n_i^c, $i = 0,1,\ldots,N$ which satisfy the first-order necessary optimality conditions (15). Let $\Omega_+ = \Omega \cap \Lambda_+$. Let $\Omega_{\min} = \Omega \cap \Lambda_{\min}$.

The set Λ_+ is fully characterised in Section 4. It contains all minimal compensators and some, but not all, non-minimal compensators with arbitrary dimensions. Therefore $\Lambda \supset \Lambda_+ \supset \Lambda_{\min}$. From Theorems 3 and 5 in Section 4, similarly, $\Omega \supset \Omega_+ \supset \Omega_{\min}$ and from Definition 6 $\Lambda \supset \Omega$, $\Lambda_+ \supset \Omega_+$ and $\Lambda_{\min} \supset \Omega_{\min}$. Theorem 1 in the companion paper presented the SDOPE and associated boundary conditions which determine the set $\Omega_{\min}$. The next theorem generalises this theorem and determines the set Ω_+ which also includes some, but not all, non-minimal optimal compensators.

Theorem 1 (Generalisation of Theorem 1 in Van Willigenburg and De Koning, 1999a). *The theorem is identical to Theorem 1 in Van Willigenburg and De Koning (1999a) when minimality of the compensator, i.e. $(\hat{x}_0, F^N, K^N, L^N) \in \Lambda_{\min}$ is replaced with pseudo-minimality of the compensator, i.e. $(\hat{x}_0, F^N, K^N, L^N) \in \Lambda_+$ and when the* *projective factorisations of Lemma 1 in the companion paper are replaced with the generalised projective factorisations in Lemma 3 of this paper. Furthermore,*

$$rank(\hat{P}_i) = rank(\hat{S}_i) = rank(\hat{P}_i\hat{S}_i) = r_i^c \leqslant n_i^c,$$
$$i = 0,1,\dots,N, \tag{18a}$$

$$\hat{x}_0 \neq 0 \Rightarrow r_0^c = 1, \quad \hat{x}_0 = 0 \Rightarrow r_0^c = 0, \quad r_N^c = 0. \tag{18b}$$

Proof. The proof is analogous to proofs presented in Haddad and Tadmor (1993) and Van Willigenburg and De Koning (2000) with the standard inverses $P_i^{2^{-1}}$, $S_i^{2^{-1}}$ replaced by the Moore–Penrose pseudo-inverses P_i^{2+}, S_i^{2+}. In general, this substitution will not yield the same results. Since Theorem 1 is restricted to the set Λ_+, relations (17) hold, which reensure the validity of the proof with the following single modification

$$G_i H_i^T = H_i G_i^T = P_i^2 P_i^{2+} = S_i^{2+} S_i^2. \tag{19}$$

In Section 4, it will appear that Eq. (19) defines precisely the generalised projective factorisation in Lemma 4 and that this factorisation implies in turn, that the compensators in Theorem 1, satisfy (17), i.e. are all elements of Λ_+. The complete proof of Theorem 1 is documented in Van Willigenburg and De Koning (1999b). □

The inequality in (18a) generalises the SDOPE to also include non-minimal optimal fixed-order compensators. In Section 5, it will be proved that r_i^c, $i = 1,2,\dots,N-1$ in (18a) can be interpreted as the dimensions of a *minimal realisation* of a compensator that satisfies Theorem 1. Therefore, the inequality in (18a) implies that non-minimal realisations within the set Λ_+, of all minimal locally optimal compensators having dimensions less than n_i^c, satisfy the first-order necessary optimality conditions. In Section 4, it is investigated as to which of all these solutions are the interesting ones in terms of the performance. From (18a) and Lemma 3, $rank(H_0) = r_0^c$, so $\hat{x}_0 = 0$ if and only if $\bar{x}_0 = 0$. Then Eq. (18b) complies with Eq. (13a).

4. Relating all compensators to the optimal projection approach

In this Section all compensators with arbitrary dimensions n_i^c, $i = 0,1,\dots,N$ are related to the optimal projection approach through generalised basis transformations, introduced in this section. Furthermore, the set Λ_+, to which Theorem 1 is restricted, is fully characterised and so are all solutions of Theorem 1, i.e. the set Ω_+. Also, it will be shown that r_i^c, $i = 1,2,\dots,N-1$ in Theorem 1 can be interpreted as the dimensions of a minimal realisation of the optimal compensator.

Lemma 4. *Λ, Ω, $\Lambda_{\min}$, $\Omega_{\min}$ are invariant under basis transformations of the compensator state-space at the discrete-time instants $i = 0,1,\dots,N$. $\Lambda_+\backslash\Lambda_{\min}$, $\Omega_+\backslash\Omega_{\min}$ are invariant under unitary basis transformations of the compensator state-space.*

Proof. For Λ the result is trivial. Let

$$A_i, \quad i = 0,1,\dots,N \tag{20a}$$

describe basis transformations at the time instants i which transform $(\hat{x}_0, F^N, K^N, L^N)$ into $(\hat{x}'_0, F'^N, K'^N, L'^N)$ given by

$$F'_i = A_{i+1}F_iA_i^{-1}, \quad K'_i = A_{i+1}K_i, \quad L'_i = L_iA_i^{-1},$$
$$\hat{x}'_i = A_i\hat{x}_i. \tag{20b}$$

If $(\hat{x}_0, F^N, K^N, L^N) \in \Omega$ it satisfies Eq. (15). Then $(\hat{x}'_0, F'^N, K'^N, L'^N) \in \Omega$ because (15) is satisfied with

$$P_i'^1 = P_i^1, \quad P_i'^{12} = P_i^{12}A_i^T, \quad P_i'^2 = A_iP_i^2A_i^T,$$
$$S_i'^1 = S_i^1, \quad S_i'^{12} = S_i^{12}A_i^{-1}, \quad S_i'^2 = A_i^{-T}S_i^2A_i^{-1}, \tag{20c}$$

where $'$ refers to values corresponding to $(\hat{x}'_0, F'^N, K'^N, L'^N)$. Then, for $\Lambda_{\min}$ and $\Omega_{\min}$ the result follows from the fact that $rank(W'_{0,i})$ and $rank(M_{i,N})$, $i = 0,1,\dots,N$ are invariant under basis transformations of the compensator state-space. For $\Lambda_+\backslash\Lambda_{\min}$, $\Omega_+\backslash\Omega_{\min}$, the result follows from (17) and the fact that for unitary matrices A_i, $S_i'^{2+} = (A_i^{-T}S_i^2A_i^{-1})^+ = A_i^{-T}S_i^{2+}A_i^{-1}$ and $P_i'^{2+} = (A_iP_i^2A_i^T)^+ = A_iP_i^{2+}A_i^T$. □

Theorem 3. *If the system has the properties $W_i > 0$, $i = 0,1,\dots,N-1$, then $rank(W'_{0,i}) = rank(P_i^2)$ $i = 0,1,\dots,N$ where $W'_{0,i}$ is the grammian, given by Eq. (11), of the compensator. Dually if the criterion has the property $R_i > 0$, $i = 0,1,\dots,N-1$ then $rank(M_{i,N}) = rank(S_i^2)$ $i = 0,1,\dots,N$ where $M_{i,N}$ is the observability grammian, given by Eq. (10), of the compensator.*

Proof. Using Kreindler and Jameson (1972), from (15j) it follows that

$$P_{i+1}^2 = (F_i + K_iC_iP_i^{12}P^{2+}_i)P_i^2(F_i + K_iC_iP_i^{12}P^{2+}_i)^T$$
$$+ K_i[C_i(P_i^1 - P_i^{12}P^{2+}_iP^{12T}_i)C_i^T + W_i]K_i^T,$$
$$i = 0,1,\dots,N-1, \qquad P_0^2 = \hat{x}_0\hat{x}_0^T, \tag{21}$$

where $P_i^1 - P_i^{12}P_i^{2+}P_i^{12T} \geqslant 0$. Given the conditions $P_i^1 - P_i^{12}P_i^{2+}P_i^{12T} \geqslant 0$, $W_i > 0$ it follows from Eqs. (11) and (21) that $rank(W'_{0,i}) = rank(P_i^2)$, $i = 0,1,\dots,N$. □

Eqs. (11) and (21) reveal that, under the conditions of Theorem 3, the propagation of P_i^2, the second moment of the compensator which is determined by the closed-loop system, and the propagation of $W'_{0,i}$, which is determined by the compensator alone, are almost identical. Dually, this holds for the propagation of S_i^2, which is determined by the dual closed-loop system and $M_{i,N}$, which is determined by the compensator alone. This similarity will be used to interpret the next, in Definitions 7–10 to be composed transformations Π and Θ, as a procedure to obtain a minimal realisation of a compensator. The transformations Π and Θ reveal connections between, and properties of, the sets Λ_+, $\Lambda_{\min}$, Ω_+ and $\Omega_{\min}$.

Definition 7 (*Generalised basis transformation Π_1*). Define the transformation $\Pi_1 C_A \to C_A^1$ which transforms the arbitrary compensator $C_A = (\hat{x}_0, F^N, K^N, L^N)$ into compensator $C_A^1 = (\hat{x}_0^1, F^{1N}, K^{1N}, L^{1N})$ with the same dimensions. The transformation is determined by P_i^2, $i = 0,1,\ldots,N$ obtained from recursions (15h)–(15j) for the compensator $C_A = (\hat{x}_0, F^N, K^N, L^N)$. Consider the following singular value decompositions of the non-negative symmetric matrices P_i^2, $i = 0,1,\ldots,N$,

$$P_i^2 = A_i D_i A_i^T, \quad i = 0,1,\ldots,N. \tag{22}$$

D_i are diagonal with the non-zero elements appearing first on the diagonal. A_i, $i = 0,1,\ldots,N$ are unitary matrices which determine unitary basis transformations,

$$\hat{x}'_i = A_i^{-1}\hat{x}_i = A_i^T \hat{x}_i \tag{23a}$$

such that,

$$P_i'^2 = \overline{\hat{x}'_i \hat{x}_i'^T} = A_i^T P_i^2 A_i = D_i = \begin{bmatrix} D_{i_r} & 0 \\ 0 & 0 \end{bmatrix},$$

$$D_{i_r} \in R^{r_i^c \times r_i^c}, \quad r_i^c = rank(P_i^2) \leqslant n_i^c, \tag{23b}$$

where $D_{i_r} \in R^{r_i^c \times r_i^c}$ are diagonal and full rank, $\hat{x}'_i$ is the compensator state after the basis transformations and $P_i'^2$ its second moment matrix. Eq. (23b) implies that the final $n_i^c - r_i^c$ components of $\hat{x}'_i$ are zero with probability one. Therefore,

$$P_i'^{12} = [P_i'^{12'} \; 0], \quad P_i'^{12'} \in R^{n_i \times r_i^c} \text{ and full rank},$$

$$i = 0,1,\ldots,N \tag{23c}$$

and, without affecting the input and the output, we may set to zero the final $n_{i+1}^c - r_{i+1}^c$ rows and final $n_i^c - r_i^c$ columns of F'_i and the final $n_i^c - r_i^c$ columns of L'_i. This is compensator $C_A^1 = (\hat{x}_0^1, F^{1^N}, K^{1^N}, L^{1^N})$.

Definition 8 (*Generalised basis transformation Π_2*). The transformation $\Pi_2 C_A \to C_A^2$ is the dual of the one in Definition 7, i.e. A_i, $i = 0,1,\ldots,N$ are now associated with singular value decompositions of S_i^2, $i = 0,1,\ldots,N$, obtained from the recursions (15e)–(15g) and the final $n_{i+1}^c - r_{i+1}^c$ rows and final $n_i^c - r_i^c$ columns of F'_i and the final $n_{i+1}^c - r_{i+1}^c$ rows of K'_i are set to zero.

Definition 9. Define the composite transformation $\Pi C_A \to C_A^2$: $C_A^2 = \Pi_2 \Pi_1 C_A$.

Definition 10 (*Generalised basis transformation Θ*). ΘC_A drops the final zero rows and columns of compensator C_A while leaving one column of zeros for F_0 and one column of zeros for L_0 if $\hat{x}_0 = 0$.

Definition 11. Two compensators are *equivalent* if their input–output behaviour over $[0, N]$ is identical.

Theorem 4. *The transformations ΠC_A and $\Theta\Pi C_A$ have the following properties*:

$$J_N(\Theta\Pi C_A) = J_N(\Pi C_A) = J_N(C_A), \tag{24a}$$

$$C_A \in \Lambda \Rightarrow \Pi C_A \in \Lambda_+, \quad C_A \in \Lambda_+ \Rightarrow \Pi C_A \in \Lambda_+, \tag{24b}$$

$$C_A \in \Omega \Rightarrow \Pi C_A \in \Omega_+, \quad C_A \in \Omega_+ \Rightarrow \Pi C_A \in \Omega_+ \tag{24c}$$

and, under the conditions of Theorem 3,

$$C_A,\ \Pi C_A,\ \Theta\Pi C_A \quad \text{are equivalent}, \tag{24d}$$

$\Theta\Pi C_A$ is a minimal realisation of C_A in the sense of Definition 4. (24e)

Proof. From Definitions 7–10 ΠC_A and $\Theta\Pi C_A$ preserve the input–output behaviour of the compensator *within the closed-loop system* and so the performance. This implies (24a). To prove (24b) observe from (23b), (23c) and (20) that the compensator $\Pi_1 C_A$ satisfies $F_i P_i^2 P_i^{2'} = F_i$, $i = 0,1,\ldots,N$ and (17b). Dually, the compensator $\Pi_2 C_A$ satisfies $S_i^{2'} S_i^2 F_i = F_i$, $i = 0,1,\ldots,N$ and (17c). Then from Definition 9, the compensator ΠC_A satisfies (17). To prove (24c), note that Π_1, Π_2 and Π consist of only unitary basis transformations and zeroing procedures. From Lemma 4, the former preserve optimality. After the unitary basis transformation (23a) of Π_1, if the optimality conditions (15) hold, given (23b), (23c), Eq. (15) is unaffected by the zeroing procedure of Π_1. Dually, the same applies to the zeroing procedure of Π_2.

To prove (24d), (24e) note that if the transformation Π_1 in Definition 7 is based on singular value decompositions of $W'_{0,i}$, instead of P_i^2, $i = 0,1,\ldots,N$ and dually if Π_2 in Definition 8 is based on singular value decompositions of $M_{i,N}$, instead of S_i^2, $i = 0,1,\ldots,N$, then, similar to the time-invariant case (Kwakernaak & Sivan, 1972), the composite transformation Π, when followed by the transformation Θ, is a procedure to obtain a minimal realisation of the compensator. Under the conditions of Theorem 3 the replacement of $W'_{0,i}$ by P_i^2 in the transformations Π_1 and dually of $M_{i,N}$ by S_i^2 in the transformation Π_2, respectively, do not alter this result. □

Theorem 5. *Under the conditions of Theorem 3, r_i^c $i = 1,2,\ldots,N-1$ in Theorem 1 represent the dimensions of a minimal realisation of the optimal compensator.*

Proof. Follows directly from the final part of the proof of Theorem 4. □

The next lemma and theorem enable a complete characterisation of the set Λ_+, to which Theorem 1 is restricted, and also of the set Ω_+ containing all solutions of Theorem 1.

Lemma 5. Elements of Λ_+, defined by Eq. (17), (15e)–(15j), have the property,

$$S_i^{2'} S_i^2 = P_i^2 P_i^{2'}, \quad i = 0,1,\ldots,N. \tag{25}$$

Proof. The result is a direct consequence of (17), (15e)–(15j). □

Theorem 6. *The set $\Lambda_+ \backslash \Lambda_{\min}$, apart from unitary basis transformations, contains minimal compensators extended with rows and columns containing only zeros. The set $\Omega_+ \backslash \Omega_{\min}$, apart from unitary basis transformations, contains minimal locally optimal compensators extended with rows and columns containing only zeros.*

Proof. From Lemma 4, $\Lambda_+ \backslash \Lambda_{\min}$ and $\Omega_+ \backslash \Omega_{\min}$ are invariant under unitary basis transformations. Then, from Lemma 4 and Eq. (23c), the unitary basis transformations (23a) of Definition 7 transform any element of $\Lambda_+ \backslash \Lambda_{\min}$ into an equivalent element of $\Lambda_+ \backslash \Lambda_{\min}$ being a minimal compensator extended with rows and columns containing only zeros. The same holds for elements of $\Omega_+ \backslash \Omega_{\min}$. □

Remark 1. In the singular case, i.e. when the conditions $W_i > 0$, $R_i > 0$ from Theorem 3 are no longer satisfied, $rank\,(W'_{0,i}) = rank\,(P_i^2)$ and $rank\,(M_{i,N}) = rank\,(S_i^2)$, $i = 0, 1, \ldots, N$ can no longer be guaranteed. In that case the transformation Π followed by Θ preserves performance, local optimality and input–output behaviour of the compensator *within the closed-loop system*, but not necessarily the input–output behaviour of the compensator alone.

5. The optimal full-order compensator, max–min compensator dimensions and numerical considerations

In this section, the finite-horizon optimal full-order compensator will be considered, which does not belong to the set Λ_+ and so falls outside the scope of Theorem 1. Using a minimal realisation of the optimal full-order compensator it is investigated which solutions of Theorem 1 are the interesting ones in terms of the performance and the minimality of the compensator. The interesting solutions turn out to be compensators with max–min dimensions, introduced in this section. The importance of max–min dimensions in choosing suitably the prescribed compensator dimensions and in solving numerically the optimal fixed-order compensation problem is explained. The choice of suitable prescribed compensator dimensions is illustrated with an example.

Theorem 7. *The finite-horizon optimal full-order compensator, i.e. with $n_i^c = n_i$, $i = 0,1,\ldots,N$, which is globally optimal, is the unique compensator that satisfies the first-order necessary optimality conditions* (15) *and the conditions $P_i^2 = P_i^{12}$, $S_i^2 = -S_i^{12}$. It belongs to the set $\Omega \backslash \Omega_+$.*

Proof. After the substitution of the optimal full-order compensator matrix expressions in the first-order necessary optimality conditions (15) these are satisfied with $P_i^2 = P_i^{12}$, $S_i^2 = -S_i^{12}$. These equalities are the ones that ensure the *global* optimality of the full-order compensator, since they state that $\hat{x}_i$, $i = 0,1,\ldots,N$ is the minimum variance estimator of x_i, $i = 0,1,\ldots,N$ and similarly for the dual compensator. Since the minimum variance estimator of x_i, $i = 0,1,\ldots,N$ is *unique*, no other compensators exist that satisfy both $P_i^2 = P_i^{12}$, $S_i^2 = -S_i^{12}$ and the first-order necessary optimality conditions.

Given the optimal full-order compensator matrix expressions and the equalities $P_i^2 = P_i^{12}$, $S_i^2 = -S_i^{12}$, SDOPE are obtained in which G_i, H_{i+1}, Ψ_i^1, Ψ_{i+1}^2, $i = 0,1,\ldots,N-1$ and $\tau_{\perp_i}$, $i = 0,1,\ldots,N$ no longer appear (Van Willigenburg & De Koning, 1999b). Due to this Eqs. (10e), (10f) of the companion paper become the well-known uncoupled estimation and control Riccati equations of full-order control. In all other cases the SDOPE are *intrinsically* coupled, except when P_i^2, S_i^2, or equivalently τ_i, $i = 0,1,\ldots,N$, are full rank. The latter is prevented by the boundary conditions (18b). □

Let n_i^m, $i = 0,1,\ldots,N$ represent the dimensions of a minimal realisation of the optimal full-order compensator. Given the global optimality of the finite horizon optimal full-order compensator it is no use choosing $n_i^c > n_i^m$ for some i.

Definition 12. The dimensions $n_i^{c_1}$, $i = 0,1,\ldots,N$ of compensator 1 are called less than the dimensions $n_i^{c_2}$, $i = 0,1,\ldots,N$ of compensator 2 if $n_i^{c_1} \leqslant n_i^{c_2}$, $i = 0,1,\ldots,N$ and if $n_i^{c_1} < n_i^{c_2}$ for at least one i.

Conjecture 1. *The minimum costs, obtainable with a minimal compensator with dimensions $n_i^c \leqslant n_i^m$, $i = 0,1,\ldots,N$, increase, if the dimensions of the minimal compensator decrease.*

Theorem 8. *Assume conjecture 1 holds. Then, if the prescribed compensator dimensions satisfy* $n_i^c \leqslant n_i^m$, $i = 0,1,\dots,N$, *Eq.* (13) *and* $n_0^c = 1$ *and* $n_N^c = 0$ *the global optimal compensator of the associated optimal fixed-order compensation problem is minimal.*

Proof. From Conjecture 1 and Theorem 5, the interesting solutions of Theorem 1 are those with *maximal* values r_i^c, $i = 1,2,\dots,N-1$, in the sense of Definition 12. These maximal values of r_i^c, $i = 1,2,\dots,N-1$ are the maximal dimensions of a minimal compensator and are therefore called max–min dimensions associated with the prescribed compensator dimensions n_i^c, $i = 1,2,\dots,N-1$. If these max–min dimensions equal the prescribed dimensions n_i^c, $i = 1,2,\dots,N-1$, this guarantees that the global optimal fixed-order compensator is minimal if, according to Definition 4, in addition $n_0^c = 1$ and $n_N^c = 0$. The latter is the case if the conditions in Theorem 8 are satisfied. □

Definition 13. Compensator dimensions that satisfy the conditions of Theorem 8 are called *max–min compensator dimensions*.

Example 1. Choice of max–min compensator dimensions.

Consider a conventional finite-horizon discrete-time LQG problem with $n_i = 4$, $m_i = 2$, $l_i = 1$, $i = 0,1,\dots,N$, $N = 5$. Note that the dimensions n_i^m, $i = 0,1,\dots,N$ of a minimal realisation of the optimal full-order compensator should satisfy (13), with n_i^c replaced by n_i^m, $n_i^m \leqslant n_i$, $i = 0,1,\dots,N$ and $n_0^m = 1$ and $n_N^m = 0$. Assume that n_i^m, $i = 0,1,\dots,N$ equal, respectively, 1,2,3,4,2,0. Then Table 1 is helpful to choose max–min compensator dimensions.

The second row, i.e. n_i^m, $i = 0,1,\dots,N$ in Table 1 represents maximum values allowed for n_i^c, $i = 0,1,\dots,N$. According to Eq. (13) the third row i.e. l_i, $i = 0,1,\dots,N$ represents the *maximum increase* allowed for n_i^c, $i = 0,1,\dots,N$ when stepping *forward* in time while the fourth row, i.e. m_i, $i = 0,1,\dots,N$ represents the *maximum increase* allowed for n_i^c, $i = 0,1,\dots,N$ when stepping *backward* in time. The fifth and sixth rows n_i^c, $n_i'^c$, $i = 0,1,\dots,N$ represent two possible choices of max–min compensator dimensions while the seventh row $n_i''^c$, $i = 0,1,\dots,N$ represents compensator dimensions that are not max–min dimensions because the increase of $n_i''^c$ from $i = 1$ to 2 exceeds the bound $l_1 = 1$. Observe from Definition 4 and Eq. (13), that when $\hat{x}_0 = 0$, the first element of the third row in Table 1, instead of l_0, is equal to $l_0 - 1$.

Remark 2. The results presented in this paper also apply to the infinite-horizon time-invariant case, when the time indices are removed everywhere, and also the boundary conditions. Eq. (13) becomes

Table 1
Choice of max–min compensator dimensions from n_i^m, l_i, m_i, $i = 0,1,\dots,N$

i	0		1		2		3		4		5
n_i^m	1		2		3		4		2		0
l_i		$\overrightarrow{+1}$		$\overrightarrow{+1}$		$\overrightarrow{+1}$		$\overrightarrow{+1}$		$\overrightarrow{+1}$	
m_i		$\overleftarrow{+2}$		$\overleftarrow{+2}$		$\overleftarrow{+2}$		$\overleftarrow{+2}$		$\overleftarrow{+2}$	
n_i^c	1		1		2		3		2		0
$n_i'^c$	1		2		3		3		1		0
$n_i''^c$	1		1		3		2		1		0

$0 \leqslant r^c \leqslant \min(n^c, n)$ and one should select n^c such that $0 \leqslant n^c \leqslant n^m$. In the infinite-horizon time-invariant case the optimal full-order compensator is usually minimal, i.e. $n^m = n$, but may not be minimal, even if the system is minimal (Yousuff & Skelton, 1984; Van Willigenburg & De Koning, 2000).

6. Conclusions

The computation of optimal fixed-order compensators based on the SDOPE should be performed as follows. First, the dimensions of a minimal realisation of the optimal full-order compensator must be computed. Given these, the prescribed compensator dimensions in the fixed-order case should be *max–min dimensions* that can be chosen based on a simple table, introduced in this paper. This guarantees that the *global optimum* is a minimal compensator, i.e. a compensator that satisfies the SDOPE in Theorem 1 with $r_i^c = n_i^c$, $i = 1,2,\dots,N-1$. In the infinite-horizon time-invariant case the full-order compensator is usually minimal while in the finite-horizon time-varying case it never is. Then the choice of max–min compensator dimensions becomes crucial. The reason is that in the finite-horizon case at the boundaries the dimensions of a minimal compensator drop in one or several time steps. In the continuous-time case this drop occurs instantaneously. Then, from the results in this paper, the problem met by Haddad and Tadmor (1993) can be overcome in a similar manner.

In deriving the SDOPE the drop of the dimension of a finite-horizon minimal compensator at the boundaries required the use of the Moore–Penrose pseudoinverse. The use of the Moore–Penrose inverse was further exploited to reveal that the optimal projection approach can be generalised, but only to partially include non-minimal compensators. Also the structure of the space of optimal compensators was revealed to a large extent.

Within the problem formulation time-varying dimensions of the system state, the input and the output are allowed. Time-varying dimensions of the input, the output and the system state, arise in digital control problems if the sampling is performed asynchronously (Van Willigenburg & De Koning, 1995), or in fault tolerant systems (Garg & Hedrick, 1993). The companion paper treated the numerical solution of the SDOPE and the generalisation to systems with white parameters for which the results of this paper also apply.

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