

## EXPERIMENTAL RESULTS OF RECEDDING HORIZON OPTIMAL CONTROL OF GREENHOUSE CLIMATE

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**Abbreviations:** RHOC - receding horizon optimal control

### Abstract

Optimal control of greenhouse climate entails the operation of the control variables such that the economic return to the grower is maximised. The basis of this approach is a model that describes the crop behaviour under the influence of the indoor climate conditions which in turn are determined by the outside weather conditions and the exerted control actions. Several simulation experiments have been reported in the literature (e.g. Gunnar et al. (1992) and van Heuven (1994)), but there is no record of any results of experiments using optimal control at an operational level in a real greenhouse. From January until June 1994 experiments have been performed in a real greenhouse with tomatoes, comparing two compartments of the same greenhouse, one controlled by a current commercial greenhouse climate control computer and one controlled by a receding horizon optimal control (RHOC) algorithm. This paper presents the optimal control problem and some preliminary results of the experiment.

### 1. Introduction

In the near future Dutch glasshouse horticulture has to meet severe governmental regulations concerning the use of energy, and the discharge of waste water, CO<sub>2</sub> and light. Moreover the market price of some major products, e.g. tomato, has been dramatically low in the past few years. Considering these problems, if Dutch horticulture wants to survive, its efficiency has to be raised. This can be done in different ways. One way is to develop new more efficient greenhouse constructions. This option definitely should be investigated, although the introduction into practice will be a long-term issue, since large investments will be needed to build new greenhouses. Another way is to use the existing greenhouses in the most efficient way. This option can be formulated as an optimal control problem: "Given the greenhouse crop system and its boundary conditions, which controls do maximise the grower's profit?". Once such a control algorithm has been developed, the introduction into practice is fairly easy, since essentially only the climate computer software needs to be changed, which is relatively cheap. In addition, optimal control will also be beneficial to renewed greenhouse designs once they are available.

When solving the optimal control problem several difficulties appear concerning the greenhouse and crop model, the criterion and the computation of the optimal controls. Using RHOC and the models described below, these problems have been solved. In simulation the RHOC-algorithm performs considerably better than the conventional control algorithm, but the proof of the promising is in the eating. So, to investigate the practical value of this approach a full scale practical experiment has been performed in a real greenhouse.

This paper reports preliminary results of a practical experiment in a greenhouse with tomatoes. The goal of this experiment is threefold: (1) testing whether the RHOC-concept in conjunction with simplified crop growth models is implementable in practice; (2) calibrating and validating the greenhouse and tomato model and (3) making a comparison with a conventional control algorithm. In this paper we will concentrate on the first goal, i.e. the implementation in practice.

## 2. Optimal Control Problem

The framework for model-based optimal control is sketched as follows. At first there is the dynamic behaviour of the greenhouse climate in response to control actions. Linked to this there is the immediate response of the photosynthesis rate to the greenhouse climate and external light. Also plant transpiration is of special interest because of its effect on air humidity. Secondly, there is the response of the crop through processing of photosynthetically assimilated material. Together with a criterion function, which reflecting the goals set by the grower, this results in an optimal control problem, which theoretically can be solved (Lewis, 1986).

### 2.1 Greenhouse climate model

The physical greenhouse model of Link ten Cate (1985), modified by Tchamitchian et al. (1992), is extended with the heating pipe temperature and the greenhouse humidity. This results in a set of five coupled first order differential equations. The greenhouse temperature  $T_g^*$  is modelled by a first order differential equation with a constant greenhouse heat capacity  $C_g$  and with heat flows from ventilation, through the cover, from the heating pipes, from the soil and from the sun.

$$\frac{dT_g^*}{dt} = \frac{1}{C_g} ((k_v k_t) (T_o - T_g^*) + k_p (T_p - T_g^*) + k_s (T_s - T_g^*) + \eta G) \quad (1)$$

The heat transfer coefficient for exchanges through the cover  $k_t$  is assumed constant, whereas the effective heat transfer coefficient  $k_v$  depends upon the window opening  $\eta_w$ , which is one of the control variables of the system. The pipe to air heat transfer coefficient  $k_p$  depends on the difference between pipe temperature  $T_p$  and greenhouse temperature  $T_g^*$ . The soil temperature  $T_s$  is modelled by a first order model with heat flows from the greenhouse, with soil to air heat transfer coefficient  $k_s$ , and from a deep soil layer of which the temperature  $T_d$  is assumed constant, with soil to deep soil heat transfer coefficient  $k_d$ .

$$\frac{dT_s}{dt} = \frac{1}{C_s} (k_s (T_g^* - T_s) + k_d (T_d - T_s)) \quad (2)$$

where  $C_s$  is the greenhouse soil heat capacity. The heat flow from the heating pipe to the greenhouse is determined by the temperature difference  $T_g^* - T_p$ . The incoming heat from the boiler is determined by the relative heating valve opening and the difference between the boiler water temperature  $T_h$  and the pipe temperature  $T_p$ .  $C_p$  denotes the heating pipe heat capacity.

$$\frac{dT_p}{dt} = \frac{1}{C_p} (k_b (T_b - T_p) + k_s (T_s - T_p)) \quad (3)$$

Here the effect of the control variable (the heating valve opening  $r_h$ ) is accounted for by the virtual heat transfer coefficient  $k_b$ . The change in CO<sub>2</sub> concentration  $C_i$  is determined by the ventilation  $\Phi_v$  and the difference between inside and outside CO<sub>2</sub> concentration  $C_i - C_o$ , the CO<sub>2</sub> dosage flux  $\Phi_i$  (control variable) and the difference between respiration  $R$  and photosynthesis  $P$ . As the crop becomes bigger, the influence of the photosynthesis and respiration on  $C_i$  increases.  $k_c$  is the CO<sub>2</sub>/CH<sub>2</sub>O conversion factor

$$\frac{dC_i}{dt} = \frac{\Phi_i (\Phi_v C_o - C_i) + \Phi_i k_c (R - P)}{V_g} \quad (4)$$

where  $V_g / A_g$  reflects the average height of the greenhouse. The ventilation flux  $\Phi_v$  together with the inside-outside absolute humidity difference  $H_f - H_o$  is a driving force of the inside absolute humidity  $H_f$ . Its two other main influences are the crop evaporation  $E$  and the condensation at the greenhouse cover  $M_c$ .

$$\frac{dH_f}{dt} = \frac{\Phi_v (\Phi_v (H_o - H_f) + E - M_c)}{V_g} \quad (5)$$

The greenhouse climate model is linked to the crop model through the state-variables  $C_i$  and  $T_g^*$ , which act as inputs to the tomato model, while  $P$ ,  $R$  and  $E$  are algebraic functions of the crop state. The greenhouse climate system and the crop system therefore interact.

### 2.2 Tomato crop model

The tomato model is a reduced version of the model described by de Koning (1994). It consists of four states which constitute a so called big leaf, big fruit model. The photosynthesis  $P$  fills the assimilate-buffer  $B$  with assimilates. From the buffer the assimilates are distributed among the stem, leaves and fruits (Fig. 1). The efficiency of converting the assimilates to fruit dry weight and leaf dry weight is reflected by  $f$  and  $v$ .

$$\frac{dB}{dt} = P - h(fG_f + vG_v)R \quad (6)$$

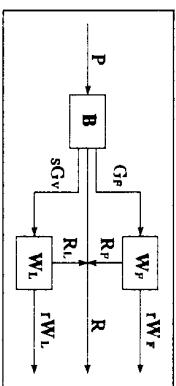


Figure 1 Simplified information flow diagram of the tomato model.

The total respiration  $R$  is the sum of the fruit respiration  $R_F$ , leaf respiration  $R_L$  and stem respiration. When the buffer is not empty the respiration uses the buffer, but when the buffer is empty the stem, leaves and fruits will respire. This transition is regulated by the buffer emptying ratio:

$$b = \frac{\mathbf{B}}{\mathbf{B} + \gamma} \quad (7)$$

where  $\gamma$  is a small constant. The potential fruit growth  $G_F$  is determined by the average temperature since anthesis  $T_f$ , the actual greenhouse temperature  $T_g$  and the fruit weight  $W_f$ . The actual fruit growth is also determined by the availability of assimilates and equals  $b G_F$ . Long-term temperature effects are partly reflected by  $T_f$

$$\frac{dT_f}{dt} = \frac{T_g - T_f}{t} \quad (8)$$

This can also be expressed as  $T_f = \frac{\int_{t_0}^t T_g dt}{t - t_0}$ , i.e.  $T_f$  acts as a temperature integral.

The fruit growth/vegetative growth ratio  $G_F / G_V$  mainly determines by the ratio of temperature and the developmental stage of the crop, which is represented by the ratio of leaf weight and fruit weight  $W_L / W_F$ . The higher  $T_g$  gets, the more fruits will be produced.

$$\frac{dW_L}{dt} = s b G_V - (1 - b) R_L - r W_L \quad (9)$$

$$\frac{dW_F}{dt} = b G_F - (1 - b) R_F - r W_F \quad (10)$$

Herein is  $s$  the ratio between stem growth and total vegetative growth. The fruit removal ratio  $r$  represents the picking of the fruits and is a function of  $W_F / W_L$  and  $t$ . It is assumed that the picking of the leaves can be described by this same ratio.

### 2.3 Criterion

Optimal greenhouse climate control may be defined as the control that satisfies best the goals set by the grower. In this paper we assume that the grower wants to maximise the operational profit. The operational profit is defined as the difference between revenues and costs. The revenues are determined by the yield and the quality of the product and the price at harvest time. The costs consist of the heating costs and the CO<sub>2</sub> dosage costs and are determined by the gas price, the CO<sub>2</sub> price and the amount of gas and CO<sub>2</sub> used. The general form of the criterion is:

$$J = \phi(\mathbf{x}(t_f), t_f) \cdot \int_{t_0}^{t_f} L(\mathbf{x}, \mathbf{u}, \mathbf{d}, t) dt \quad (11)$$

In this  $\mathbf{x}$  are the states,  $\mathbf{u}$  the controls and  $\mathbf{d}$  the weather,  $t_0$  is the initial time,  $t_f$  the final time and  $\phi(\mathbf{x}(t_f), t_f)$  represents the revenues at the final time  $t_f$ .

$$\phi(\mathbf{x}(t_f), t_f) = p_F(t_f) \mathbf{W}_F(t_f) \quad (12)$$

where  $p_F(t_f)$  is the fruit price at  $t_f$ ,  $I(x, u, d, t)$  equals the difference between the revenues and the costs at time  $t$ , decreased by the penalty function  $P(\mathbf{x}, t)$ .

$$L(\mathbf{x}, \mathbf{u}, \mathbf{d}, t) = p_F t \mathbf{W}_F - p_H k_p (\mathbf{T}_p - \mathbf{T}_g) - P(\mathbf{x}, t) \quad (13)$$

where  $p_C$  and  $p_H$  are the CO<sub>2</sub> and heating prices. The real profit is easy to calculate when the tomato price is constant, but is difficult to calculate when it is time varying. This is so, because tomato biomass produced now may be harvested at any time during the coming six weeks. In this experiment the tomato price is assumed to be constant. The penalty function  $P(\mathbf{x}, t)$  conveys physiological knowledge which is not taken into account in the plant model and is used to keep the greenhouse climate within certain boundaries. When the climate stays within these boundaries the penalty function equals zero, otherwise the penalty function becomes negative:

$$P(\mathbf{x}, t) = \begin{cases} \alpha^T(\mathbf{x}, b_l(t)) & \text{if } \mathbf{x} \leq b_l(t) \\ 0 & \text{if } b_l(t) < \mathbf{x} \leq b_u(t) \\ \alpha^T(\mathbf{x}, b_u(t)) & \text{if } b_u(t) < \mathbf{x} \end{cases} \quad (14)$$

where  $b_l$  and  $b_u$  are the lower and upper bounds for the states  $\mathbf{x}$  and  $\alpha$  determines the severity of the bounds. In order to keep the states  $\mathbf{x}$  within its bounds,  $\alpha$  must not be too small. The physiological bounds are no hard bounds, so  $\alpha$  must not be too big either, which facilitates the optimisation also. In addition to physiological knowledge, the penalty function can also pertain to possible governmental measures, for instance a tax on CO<sub>2</sub>-emission or a maximum energy consumption per square meter.

### 2.4 Receding horizon optimal control

Optimal control is essentially open loop and the computations can be done off line. As long as the simulated system behaviour differs only slightly from the actual system behaviour, this approach will work. In practice however deviations will occur, since the greenhouse and crop models are not perfect and the weather is difficult to predict. Feedback is therefore required.

In RHOC, a finite horizon open-loop optimal control problem, with initial values based on actual measurements, is solved at each time step, but only the first value of the computed control is applied at the next time step. The computation is repeated at the next time step, moving the horizon one step up. This results in a feedback strategy which also can be applied to nonlinear systems (Mayne and Michalska, 1990). Short term optimisations are allowed, because a two time-scale decomposition of the complete problem is possible. This means that the complete problem can be divided into a slow,

long term subproblem and a fast, short term subproblem. The solution of the slow subproblem, which can be solved independently, can be used as an input for the fast subproblem (van Henten, 1994).

To get maximum profit for a whole growing season, the tomato production must be guaranteed throughout the growing season. On the short term it can be optimal to produce as many fruits and as few leaves as possible, but in this way the future production is not guaranteed and on the long term it is not optimal. As the solution for the long term problem for this particular tomato model was not yet available at the time the experiment started, we have ignored the problem of guaranteeing crop production over the whole growing season. The optimisation was performed over a moving time window of 60 minutes, using both the fast greenhouse climate and the crop model and assuming that only the fruits represent economic value. The consequences of this assumption will be discussed later.

Simulations show that in the case of lettuce, for short term optimisations and using RHOC, the best results are obtained with an optimisation horizon of one hour combined with the "lazy man" weather predictions (Tap et al., 1995). The "lazy man" weather predictions means that during the period  $t_n \rightarrow t_{n+V}$  the weather is assumed to be constant and equal to the weather at  $t_n$ . Thanks to the relatively short optimisation horizon  $N$ , the computation time is less than the control time-step, so the RHOC-algorithm has the desired property that it can be implemented in real time. For lettuce the performance criterion obtained in simulation was only + 15% below the hypothetical optimum using a priori perfect weather knowledge.

### 3. Preliminary Field Test

#### 3.1 Description

In this experiment tomato plants are grown simultaneously, in two sections of the same greenhouse. During the period represented in figure 2 the tomatoes were in their reproductive phase. One section of the greenhouse is controlled by the RHOC-algorithm (with several interruptions for technical reasons; during these periods conventional control was applied), the other section is controlled by a conventional control algorithm. The heating valve opening as well as the window opening and the  $\text{CO}_2$  dosage can have any value between their minimum and maximum value.

	Conventional controller	RHOC
min $T_e$ night	17 °C	15 °C
min $T_e$ day	18 °C	17 °C
max $T_e$ night	20 °C	25 °C
max $T_e$ day	22 °C	25 °C
$H_R$ night	75 %	
$H_R$ day	80 %	
min. $H_R$		65 %
max. $H_R$		90 %
$\text{max. } C_i$		1000 ppm

Table 1 Settings of the optimal and the conventional controller

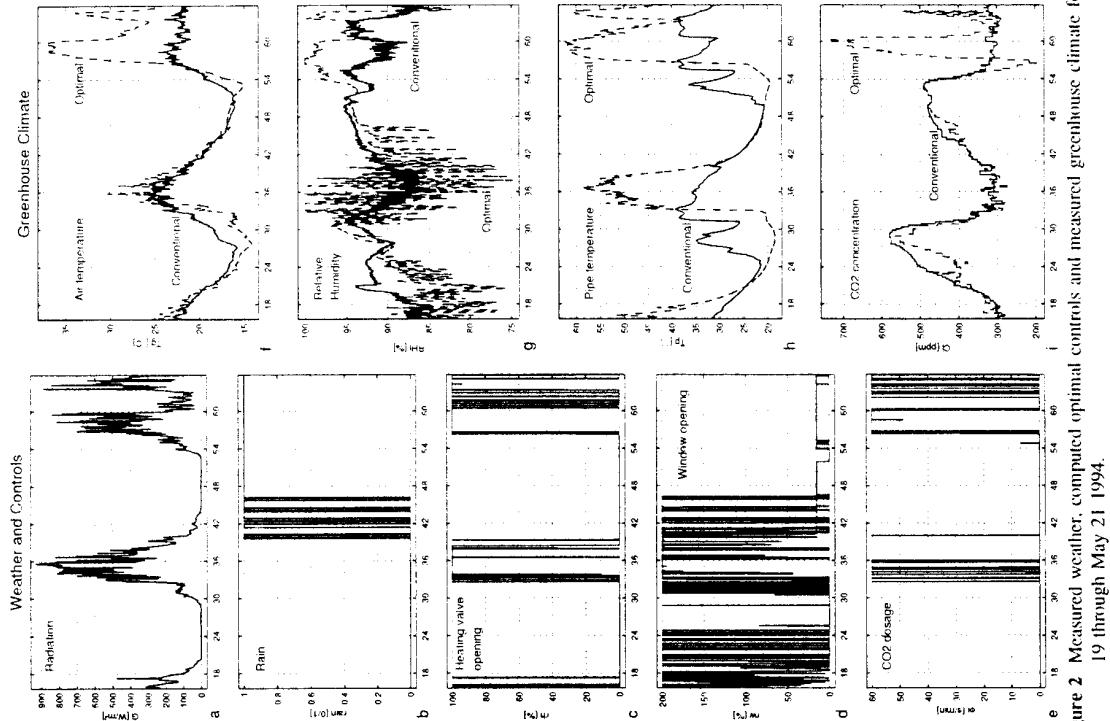


Figure 2 Measured weather, computed optimal controls and measured greenhouse climate for May 19 through May 21 1994.

During the time span described in figure 2 the  $\text{CO}_2$ -dosage of the conventional controller was turned off. The time step of the optimal control is one minute and the optimisation horizon is 60 minutes. The outside and inside climate are measured every minute and cropmeasurements are done once a week. The computation of the controls is based on Pontryagin's maximum principle using a first order gradient method (Lewis, 1986).

### 3.2 Results

Figure 2 gives the weather measurements, the computed optimal controls and the greenhouse climate measurements of May 19-15.00 through May 21-17:00. Figure 2.a shows the global radiation  $G$  which is the main weather influence upon the greenhouse climate. Figure 2.b indicates whether it rains or not. When it does not rain  $\text{train} = 0$ , the maximum window opening is 200% (0-100% for the lee side window, 100-200% for the windward side window). When it rains  $\text{train} = 1$ , the maximum window opening is limited to 15%. Figure 2.c through 2.e show the computed optimal controls. In fact  $r_h$ ,  $r_w$  and  $\Phi_t$  are not the real control variables. But because they have comparatively fast dynamics (both  $r_w$  and  $r_h$  have a response-time of about 2 minutes) they can be considered the control variables. Finally figures 2.f through 2.i show the greenhouse climate variables. The dashed lines represent the "optimal" climate, whereas the solid lines represent the "conventional" climate.

To investigate the implementability of RHOC in the greenhouse practice we will not focus on the differences between the optimal and conventional control. In the following discussion, three time periods will be investigated: firstly the day ( $G > 0$ ), secondly the night ( $G = 0$ ) and thirdly the time it rains ( $\text{train} = 1$ ).

Almost at the end of the first and second day ( $15 < \text{time} < 20$  h) and  $33 < \text{time} < 40$  h) the optimal  $T_p$  is higher and the optimal  $r_w$  is higher and fluctuates strongly. As a result the optimal  $T_g$  is almost equal but fluctuating, whereas the optimal inside relative humidity  $H_{Ri}$  is lower and fluctuates much more than the conventional  $H_{Ri}$ . The criterion function is the difference between the operating costs and the revenues of the tomato fruits (eq. 13). So when there are assimilates available ( $B > O \rightarrow b > O$ ) and when the heating costs are less than the fruit growth profit, RHOC raises  $T_g$ , which will be done by raising  $T_p$ . At first this lowers  $H_{Ri}$ , since the relative humidity is a function of the vapour concentration and the temperature. On the other hand, an increase in  $T_g$  increases the transpiration, so, after some time, when the windows are closed and the vapour accumulates, it causes  $H_{Ri}$  to rise. So, to keep the  $H_{Ri}$  within its boundary values, the windows will be opened to prevent this. Then we see that both the temperature and relative humidity drop far below its boundary values and we conclude that the windows are opened too far. Presumably this happens because the ventilation model underestimates the true ventilation. To raise  $H_{Ri}$  again, the windows are closed, but also too far. And hence  $H_{Ri}$  becomes too high and the whole cycle starts again. This cycle ends when the heating costs don't counterbalance the profit of the extra fruit growth any more and the heating is turned off. This happens at the end of the day when  $B \approx 0$  and  $P \approx 0$ .

In fact there are three reasons to heat: (1) to regulate the distribution of assimilates, (2) to keep  $T_g$  within its boundaries and (3) to keep  $H_{Ri}$  within its boundaries. During the end of the day period, the night period and the start of the day period ( $18 < \text{time} < 32$  h and  $40 < \text{time} < 56$  h)  $B \approx O$ ,  $T_g$  is over its lower limit and  $H_{Ri}$  can be kept within its limits by manipulating the window opening, which is the preferred way to control humidity, since there are no costs connected to window movements in the criterion. So there is no reason to heat.

During the third day it is dry, but the rain meter used by the RHOC goes wrong and indicates rain by mistake, so the maximum window opening is reduced to 15%. Since the conventional controller uses an independent rain meter, this doesn't influence the conventional controller. The reduced window opening limits the ventilation flux. RHOC seizes this opportunity of simultaneous radiation and limited ventilation flux to stimulate photosynthesis by dosing  $\text{CO}_2$ , so the  $\text{CO}_2$ -concentration exceeds 700 ppm. The limited ventilation flux also causes a high  $H_{Ri}$ . In order to reduce  $H_{Ri}$ ,  $T_g$  is increased by turning on the heating. Together with the limited ventilation flux and a global radiation of about 500 W/m<sup>2</sup>, this causes  $T_g$  to exceed 35 °C.

At the end of the first and second night and at the start of the second and third day ( $25 < \text{time} < 33$  h and  $50 < \text{time} < 56$  h), the optimal greenhouse and pipe temperature are lower than the conventional ones. The conventional controller is heating because it wants to raise the fruit temperature in order to prevent condensation on the fruits. This might occur when the sun starts to shine and the absolute humidity inside the greenhouse rises while the fruit temperature is still relatively low. The models used for the RHOC do not account for this effect.

The computed controls have a bang-bang character and fluctuate a lot. For the  $\text{CO}_2$  control this is no problem, but for the heating valve position and especially for the window opening it is. The fast fluctuations cause a turbulent atmosphere which is considered to be undesirable for the tomato crop. Often in simulation the controls also had a bang-bang character, but with less switching instants, so a proper calibration of the model presumably will solve the greatest part of the problem.

Probably, some of the above-mentioned problems concerning the heating can be resolved by first solving the slow subproblem for the entire season and subsequently using its solution to adapt the short term criterion function (eq. 5.149 van Heeten, 1994). The result is a short term criterion function which values not only the fruits, but all crop states. So the head demand probably will spread over the day and will probably differ for different days in the growing season. By this method we expect the fruit production to be continuous throughout the growing season.

### 4. Discussion and Conclusions

The results presented here show a number of interesting features and a number of difficulties of optimal control. The interesting aspect is that the RHOC as implemented exploits the opportunities offered by the actual weather and the greenhouse dynamics. In fact every moment it weighs up the costs and the revenues. If properly implemented this form of control is expected to bring considerable profits. The following difficulties that came to light need to be resolved:

- technical: safeguarding against sensor failure is needed (as in classical control).
- Optimisation: the rapid switching of the windows is undesirable and probably is not necessary. A possible cure is to put a small cost on window operation.
- models: calibrating, validating and if necessary adapting the greenhouse and crop model. At first this should be done off-line. Later an on-line calibration algorithm should be implemented.

- goal function: short term control should have a marginal value for each plant state, to be derived from long term, seasonal optimisation using nominal weather.
- care should be taken to determine the penalty functions so that they properly handle the unmodelled aspects of the plant behaviour. This can be done interactively by a plant expert using simulation.

The results of this (to our knowledge) first experiment using RHOC optimal control at an operational level in a real greenhouse show that RHOC is technically feasible. The expectation is that after solving of the above problems RHOC is also implementable

from a crop and from an economic point of view. Future research will verify this expectation by means of a repetition of the experiment with an adapted version of the RHOC algorithm.

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## THE EFFECT OF ELECTRO-HYDRAULIC DEVICES ON THE PERFORMANCE AND STABILITY OF A FEEDBACK SYSTEM

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#### Abstract

Two different electro-hydraulic compensators which must attenuate horizontal flexible deformations of a slender beam are designed. The performance of the second controller which incorporates only the dynamics of the beam, servovalve and cylinder, is strongly inferior to that of the first controller which takes into account the dynamics of the beam and the complete hydraulic system (hydraulic lines, valves, accumulators and hydraulic cylinder).

#### 1. Introduction

During laboratory experiments with fast electro-hydraulic compensators, which must be able to attenuate flexible deformations on an slender beam, it was found that neglected dynamics of the hydraulic system could excite flexible modes of the beam instead of controlling them.

The objective of this paper is to show through detailed mathematical models, how the hydraulic devices (hydraulic lines, valves, accumulators and cylinder) affect the high speed performance and the robust stability of fast electro-hydraulic feedback systems. It will be demonstrated that it is still possible to develop controllers with robust stability and high performance by employing suitable design models. These models must be derived from a detailed evaluation model, that incorporates the dynamics of the hydraulic system, with appropriate truncation techniques.

#### 2. Description of the plant

A slender hollow beam of four meter length is clamped onto a cradle which is driven along a horizontal inertial frame by a translational hydraulic cylinder with servovalve (figure 1). The inside and outside cross sections of the beam are resp. 16 mm by 56 mm and 20 mm by 60 mm. The two lowest horizontal natural frequencies of the beam measure 9.2 rad/s and 56.2 rad/s. Horizontal elastic deformations of the beam, induced by longitudinal accelerations of the cradle are attenuated by a second identical hydraulic cylinder with servovalve attached to the cradle and the beam at 1 m from the clamped end and parallel to the inertial frame.

The hydraulic lines and hoses are modelled with the lumped fluid circuit theory, because their length is reasonably short ( $< 10$  m) (Walton, 1989). The four-way servovalve is of the open centre type (symmetrically underdrapped) with linear ports and a nominal rating of 3 l/min at a valve pressure drop of 70 bar. It is connected with a double-acting double-rod linear actuator which has a stroke of 10 cm, a net area of 2.9 cm<sup>2</sup> and an estimated viscous friction coefficient set equal to 350 Ns/m.

Two nitrogen-loaded accumulators each with a content of 1 l are placed in the pressure line just before the servovalve and in the return line just beyond the servovalve. The former provides a high flowrate of fluid when required while the latter is used as a pulsation damper. The pressure relief valve between the pump and the first accumulator is set to off-load at 100