State-Space Example 1

Temperature in a room

- System border
  De systeemgrens wordt bepaald door wat we wel en wat we niet als onderdeel van het systeem beschouwen.

- The System equation
  \[
  \frac{dT_{th}(t)}{dt} = c_1(T_h(t) - T_b(t)) + c_2v(t) \quad \text{of} \quad \frac{dT_{tb}(t)}{dt} = c_1(T_b - T_b) + c_2v
  \]

  \(t\) time
  \(T_h\) room temperature
  \(T_b\) outside temperature
  \(c_1, c_2\) System constants
  \(\frac{dT_{th}}{dt}\) Changes per time unit of the room temperature
Questions

1) How do we obtain the system equations and constants?
2) What are the units of the quantities appearing in the model? Are they unique?
3) What are the system inputs? Which are control inputs and which are external inputs?
4) How does the system behave, what exactly determines the system behavior?
5) Can we calculate the system behavior and if so what information do we need for this?

Answers

1) Physical knowledge described by physical laws. First principles.
   \( c_1 \) determined by:
   - heat conduction of the walls
   - size of walls
   - heat capacity of the room -> volume of room, heat capacity of air
   No heat conductance through the roof? Floor 1 and 2 neglected?

   \( c_2 \) determined by the efficiency of the heater and the heat capacity of the room

   \[
   c_1 = \frac{w_{\text{wall}} \; opp_{\text{wall}}}{\text{volume}_{\text{room}} \; sw_{\text{air}}} \quad [1/s] = \left[ \frac{J}{(Km^2s)} \right] \quad \left[ m^2 \right] \quad \left[ J/(m^3) \right] \\
   c_2 = \frac{we_{\text{heater}}}{\text{volume}_{\text{room}} \; sw_{\text{air}}} \quad [K/J] = \left[ \frac{J}{K} \right] \quad \left[ m^3 \right] \quad \left[ J/(Km^3) \right]
   \]

   \( w_{\text{wall}} \) : \( ((J/s)/(K/m^2))=\left[ J/(Km^2s) \right] \) (Joule per second) per Kelvin temperature difference per square meter
   \( we_{\text{verwarming}} \) : \( [J/J] \): Joules of heat supplied by the heater per Joule electrical energy consumed by the heater
   \( sw_{\text{air}} \) : \( [J/K/m^3]=\left[ J/(Km^3) \right] \) Joule per Kelvin temperature increase per cubic meter.

   \[
   \frac{dT_h}{dt} = c_1 \; (T_b - T_h) + \; c_2 \; v \\
   [K/s] = [1/s] \quad [K] \quad + [K/J][J/s]
   \]

2) \( \frac{dT_h}{dt} : [K/s] \) Temperatuurverandering van de huiskamer per seconde.
   \( v : [J/s] \) Joule per seconde = Watt is het momentaan vermogen dat de verwarming verbruikt (groter dan wat hij aan warmte afgeeft).
3) \( T_b \) External input that cannot be influenced but is determined by external factors. \( v \) control input that can be chosen freely at each time within certain bounds.

4) The system behavior equals the time evolution \( T_h(t) \) of the room temperature. The room temperature is the only system variable.

5) Yes. Consider \( T_h(t) \), \( t_0 \leq t \leq t_f \).

\[
\frac{dT_h(t)}{dt} = \lim_{\Delta t \to 0} \frac{T_h(t + \Delta t) - T_h(t)}{\Delta t} = c_1 (T_h(t) - T_h(t)) + c_2 v(t) \quad (A1)
\]

Presume \( T_h(t_0), T_h(t_0), v(t_0) \) are known. Take \( t = t_0 \) and a small value \( \Delta t > 0 \). Then from the system equation

\[
\frac{T_h(t_0 + \Delta t) - T_h(t_0)}{\Delta t} = c_1 (T_h(t_0) - T_h(t_0)) + c_2 v(t_0) \quad \text{or,} \quad (A2)
\]

\[
T_h(t_0 + \Delta t) - T_h(t_0) = \Delta t [c_1 (T_h(t_0) - T_h(t_0)) + c_2 v(t_0)] \quad \text{or,} \quad (A3)
\]

\[
T_h(t_0 + \Delta t) = T_h(t_0) + \Delta t [c_1 (T_h(t_0) - T_h(t_0)) + c_2 v(t_0)] \quad (A4)
\]

On the right everything is known. Define,

\[
t_1 = t_0 + \Delta t \quad (A5)
\]

Presume \( T_h(t_1) = T_h(t_0 + \Delta t) \) and \( v(t_1) = v(t_0 + \Delta t) \) are known. Notice that \( T_h(t_1) = T_h(t_0 + \Delta t) \) has just been computed. Now from the system equation

\[
\frac{T_h(t_1 + \Delta t) - T_h(t_1)}{\Delta t} = c_1 (T_h(t_1) - T_h(t_1)) + c_2 v(t_1) \quad \text{or,} \quad (A6)
\]

\[
T_h(t_1 + \Delta t) - T_h(t_1) = \Delta t [c_1 (T_h(t_1) - T_h(t_1)) + c_2 v(t_1)] \quad \text{or,} \quad (A7)
\]

\[
T_h(t_1 + \Delta t) = T_h(t_1) + \Delta t [c_1 (T_h(t_1) - T_h(t_1)) + c_2 v(t_1)] \quad (A8)
\]

Again everything on the right is known. Summarizing we can calculate through this so called Euler integration method \( T_h(t), t_0 \leq t \leq t_f \) for a FINITE number of time instants \( t \). To do this \( T_h(t_0), T_h(t), v(t), t_0 \leq t \leq t_f \) must be known. The smaller \( \Delta t \) the more accurate the computation but the larger the number of time steps and the associated computation time. Remark: for very small values of \( \Delta t \) rounding errors start to dominate and destroy the accuracy of the computation. How can we determine experimentally whether \( \Delta t \) is chosen sufficiently small?