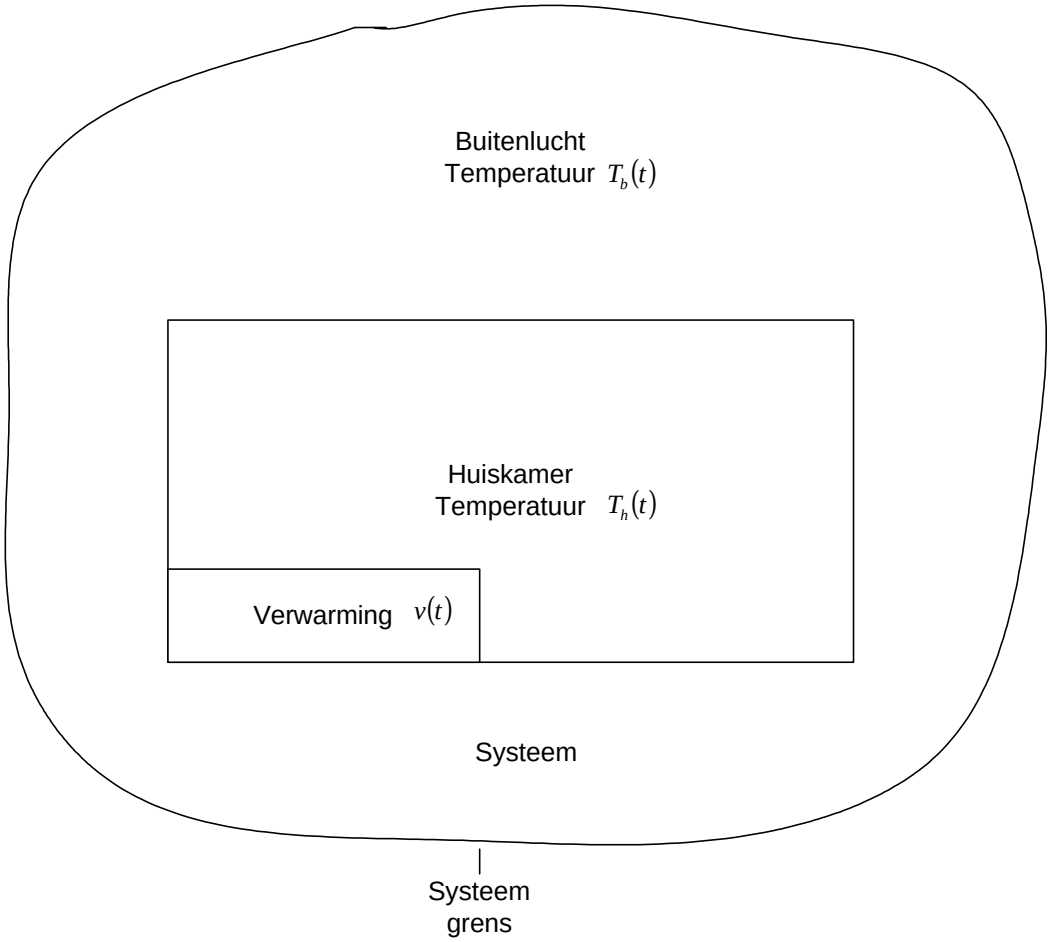
State-Space Example 1

Temperature in a room



- **System border**

  De systeemgrens wordt bepaald door wat we wel en wat we niet als onderdeel van het systeem beschouwen.

- **The System equation**

$$\frac{dT_h(t)}{dt} = c_1\left(T_b(t) - T_h(t)\right) + c_2 v(t) \quad \text{of} \quad \frac{dT_h}{dt} = c_1\left(T_b - T_h\right) + c_2 v$$

t	time
T_h	room temperature
T_b	outside temperature
c_1, c_2	System constants
$\frac{dT_h}{dt}$	Changes per time unit of the room temperature

Questions

1) How do we obtain the system equations and constants?

2) What are the units of the quantities appearing in the model? Are they unique?

3) What are the system inputs? Which are control inputs and which are external inputs?

4) How does the system behave, what exactly determines the system behavior?

5) Can we calculate the system behavior and if so what information do we need for this?

Answers

1) Physical knowledge described by physical laws. First principles.
c_1 determined by:
heat conduction of the walls
size of walls
heat capacity of the room -> volume of room, heat capacity of air
No heat conductance through the roof? Floor 1 and 2 neglected?

c_2 determined by the efficiency of the heater and the heat capacity of the room

$$c_1 = wg_{\text{wall}} \; opp_{\text{wall}} \; / \left(volume_{\text{room}} sw_{\text{air}} \right)$$
$$[1/s] = [J/(Km^2 s)] \; [m^2] \; / \left([m^3] \; [J/(Km^3)] \right)$$

$$c_2 = we_{\text{heater}} / \left(volume_{\text{room}} sw_{\text{air}} \right)$$
$$[K/J] = [J/J] \; / \; [m^3] \; [J/(Km^3)]$$

$wg_{\text{wall}} : [((J/s)/K)/m^2] = [J/(Km^2 s)]$ ((Joule per second) per Kelvin temperature difference) per square meter
$we_{\text{verwarming}} : [J/J]$: Joules of heat supplied by the heater per Joule electrical energy consumed by the heater
$sw_{\text{air}} : [(J/K)/m^3] = [J/(Km^3)]$ Joule per Kelvin temperature increase per cubic meter.

2)
$$\frac{dT_h}{dt} = c_1 \; (T_b - T_h) + c_2 \; v$$
$$[K/s] = [1/s] \; [K] + [K/J][J/s]$$

$\frac{dT_h}{dt} : [K/s]$ Temperatuurverandering van de huiskamer per seconde.
$v : [J/s]$ Joule per seconde = Watt is het momentaan vermogen dat de verwarming verbruikt (groter dan wat hij aan warmte afgeeft).

3) T_b External input that cannot be influenced but is determined by external factors. v control input that can be chosen freely at each time within certain bounds.

4) The system behavior equals the time evolution $T_h(t)$ of the room temperature. The room temperature is the only system variable.

5) Yes. Consider $T_h(t),\ t_0 \le t \le t_f$.

$$\frac{dT_h(t)}{dt} = \lim_{\Delta t \to 0} \frac{T_h(t+\Delta t) - T_h(t)}{\Delta t} = c_1(T_b(t) - T_h(t)) + c_2 v(t) \tag{A1}$$

Presume $T_h(t_0),\ T_b(t_0),\ v(t_0)$ are known. Take $t = t_0$ and a small value $\Delta t > 0$. Then from the system equation

$$\frac{T_h(t_0+\Delta t) - T_h(t_0)}{\Delta t} \approx c_1(T_b(t_0) - T_h(t_0)) + c_2 v(t_0) \text{ or,} \tag{A2}$$

$$T_h(t_0+\Delta t) - T_h(t_0) \approx \Delta t[c_1(T_b(t_0) - T_h(t_0)) + c_2 v(t_0)] \text{ or,} \tag{A3}$$

$$T_h(t_0+\Delta t) \approx T_h(t_0) + \Delta t[c_1(T_b(t_0) - T_h(t_0)) + c_2 v(t_0)] \tag{A4}$$

On the right everything is known. Define,

$$t_1 = t_0 + \Delta t \tag{A5}$$

Presume $T_b(t_1) = T_b(t_0+\Delta t)$ and $v(t_1) = v(t_0+\Delta t)$ are known. Notice that $T_h(t_1) = T_h(t_0+\Delta t)$ has just been computed. Now from the system equation

$$\frac{T_h(t_1+\Delta t) - T_h(t_1)}{\Delta t} \approx c_1(T_b(t_1) - T_h(t_1)) + c_2 v(t_1) \text{ or,} \tag{A6}$$

$$T_h(t_1+\Delta t) - T_h(t_1) \approx \Delta t[c_1(T_b(t_1) - T_h(t_1)) + c_2 v(t_1)] \text{ or} \tag{A7}$$

$$T_h(t_1+\Delta t) \approx T_h(t_1) + \Delta t[c_1(T_b(t_1) - T_h(t_1)) + c_2 v(t_1)] \tag{A8}$$

Again everything on the right is known. Summarizing we can calculate through this so called Euler integration method $T_h(t),\ t_0 \le t \le t_f$ for a FINITE number of time instants t. To do this $T_h(t_0)$, $T_b(t),\ v(t)$, $t_0 \le t \le t_f$ must be known. The smaller Δt the more accurate the computation but the larger the number of time steps and the associated computation time. Remark : for very small values of Δt rounding errors start to dominate and destroy the accuracy of the computation. How can we determine experimentally whether Δt is chosen sufficiently small?