TP-3-5

TRUE DIGITAL TRACKING FOR AN ORTHOGONAL ROBOT MANIPULATOR

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Summary

For a general continuous time linear stochastic system with piecewise constant control, the continuous time tracking problem, where the criterion is quadratic, is transformed to a continuous time linear stochastic regulator problem, which in turn is transformed to the so called equivalent discrete stochastic regulator problem of which the solution is known. The result constitutes a "true" digital feedback controller, which for instance can be used to control a orthogonal robot.

Introduction

An orthogonal robot is a continuous linear time invariant system if we ignore flexibility and non linear friction.Digital controllers for robot manipulators very often, if not always, use zero order hold circuits. In these cases control is piecewise constant.

A well known approach in designing a digital controller for such a continuous linear system is to transform the continuous time system with piecewise constant control to a so called equivalent discrete system (EDS), which describes the continuous time system behaviour at the sampling instants. Using this EDS the designer chooses a discrete time criterion, to arrive at a digital control algorithm. Choosing the discrete time criterion and the sampling time is often stated to be a problem, considering the continuous time system behaviour. This approach is sometimes called true digital control [6].

Controlling a continuous time system one is interested in the continuous time system behaviour, so it is natural to use continuous time criteria. In case of a continuous time linear quadratic stochastic regulator problem it is possible to transform the complete control problem, i.e. the continuous system with piecewise constant control and the continuous cost criterion, into a so called equivalent discrete stochastic regulator problem (EDSRP)[1,2,4]. The EDSRP consists of the EDS mentioned earlier and a discrete quadratic cost criterion which is now the equivalent of the continuous time cost criterion. It would be preferable to call this approach true digital control since now the continuous time system behavior is explicitly considered. The solution of this EDSRP is known for time invariant systems [1].

The linear quadratic tracking problem with piecewise constant control can be transformed into a linear quadratic regulator problem with piecewise constant control and time invariant system matrices if the trajectory can be modeled as the output of a linear time invariant system.

In chapter one of this paper the continuous time linear tracking problem for an orthogonal robot is presented. Chapter two presents the general form of the EDSRP and its solution. In chapter three and four the tracking problem is transformed into a regulator problem with time invariant system matrices by approximating the trajectory with a finite number of sine functions. The resulting EDSRP and its closed loop solution, which may serve as a digital control algorithm, are considered. The solution can also be used to calculate an open loop optimal piecewise constant control. This open loop solution can be used together with a separately designed digital perturbation controller.

1. Digital tracking problem for an orthogonal robot

An orthogonal robot mechanism consists of three axes moving perpendicularly. If we ignore flexibility of the robot mechanism there is no coupling between the movements of each link. Each link is assumed to be actuated by a current controlled DC motor, through a gearbox, together refered to as the drive system. If we ignore the fast dynamics of the current controller, besides flexibility, play, and non linear friction acting on the mechanism and drive system, the dynamics of each link can be modeled independently by

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -m1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ m2 \end{bmatrix} u + \begin{bmatrix} 0 \\ m3\ \mathrm{sign}(x) \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \quad (1)$$

In (1) x represents the link translation with respect to a reference position, $\dot{x}$ link speed, and $\ddot{x}$ link acceleration. The control variable u represents the motor current. The model parameter m1 represents viscous friction acting on the mechanism and drive system, m2 depends on the actuator and gearbox characteristics, and m3 represents dry friction acting on the mechanism and drive system. The dry friction term in (1) is known once sign(x) is known and can be compensated by adding m3*sign(x)/m2 to the control. Since x is a state variable in (1) we will measure x or reconstruct it, so we will always be able to compensate on-line for dry friction. The compensation of dry friction will be ommitted from the control problem, which in this case takes the desired linear form. Finally w_1 and w_2 represent white noise processes that can be used to model different kind of uncertainties.

Since there is no coupling between the links the digital tracking problem for the orthogonal robot can be split up into three seperate digital tracking problems, one for each link. The digital tracking problem for each link has the following

general form
Given the linear time invariant stochastic system

$$\dot{\underline{x}} = A\,\underline{x} + B\,\underline{u} + \underline{w} \tag{2}$$

where $\underline{w}$ is a white noise vector stochastic process

find the piecewise constant control

$$u(t) = u(kT_s)\ ,\ kT_s \le t < (k+1)T_s \quad ,k=0,1,\dots,N-1 \tag{3}$$

that minimizes

$$E\{\int_{t_0}^{t_f} (\underline{x}-\underline{x}_0)^T Q(\underline{x}-\underline{x}_0) + \underline{u}^T R \underline{u}\,dt\}$$
$$+(\underline{x}(t_f)-\underline{x}_0(t_f))^T H(\underline{x}(t_f)-\underline{x}_0(t_f)) \tag{4}$$

with $t_f = NT_s$

and T_s the sampling time fixed and known.

$\underline{x}_0$ represents the desired state trajectory, in this case following from the desired robot movement, which has to be known as a function of time. Notice that besides deviations from the desired trajectory the magnitude of the control is weighted in the costcriterion.

2. The Equivalent Discrete Stochastic Regulator Problem (EDSRP) and its solution.

Given a linear time invariant continuous stochastic system (2) with piecewise constant control (3) then the linear quadratic stochastic regulator problem is concerned with finding the control sequence (3) that minimizes the continuous criterion

$$E\{\int_0^{t_f} \underline{x}^T Q \underline{x} + \underline{u}^T R \underline{u}\ dt + \underline{x}\ (t_f) H \underline{x}(t_f)\} \tag{5}$$

The continuous problem ((2),(3),(5)) with constraints (3) on the continuous control can be transformed into a discrete time stochastic regulator problem with unconstrained control [1,2]. We state the following result. Given the linear quadratic regulator problem with piecewise constant control ((2),(3),(5)) the EDSRP takes the following form

Find the discrete control sequence

$$\underline{u}_k \quad k=0,1,\dots\dots,N-1 \tag{6}$$

that minimizes

$$E\{\sum_{k=0}^{N-1} \underline{x}_k^T Q \underline{x}_k + \underline{u}_k^T R \underline{u}_k + \underline{x}_k^T M \underline{u}_k + \underline{x}_N^T H \underline{x}_N\} + \sum_{k=0}^{N-1} \gamma \tag{7}$$

subjected to

$$\underline{x}_{k+1} = \Phi \underline{x}_k + \Gamma \underline{u}_k + \underline{v} \tag{8}$$

The so called equivalent discrete system (EDS) which describes the system behavior (2) at the sampling instants where

$$\Phi = exp(AT_s) \tag{9}$$

$$\Gamma = \int_0^{T_s} \Phi(t) B dt \tag{10}$$

$$Q = \int_0^{T_s} \Phi^T(t) Q \Phi(t) dt \tag{11}$$

$$R = \int_0^{T_s} [R + \Gamma^T(t) Q \Gamma(t)] dt \tag{12}$$

$$M = \int_0^{T_s} \Phi^T(t) Q \Gamma(t) dt \tag{13}$$

$$\gamma = \int_0^{T_s} tr[V(t)Q] dt \tag{14}$$

with

$$V(t) = \int_0^{T_s} \Phi(s) W \Phi^T(s) ds \tag{15}$$

where W is the covariance matrix of the vector stochastic process $\underline{w}$. The discrete time white noise $\underline{v}$ is defined by covariance matrix $V(T_s)$.

The resulting linear quadratic stochastic regulator problem differs from the standard form in that a crossterm $2\underline{x}^T M u$ appears in the cost criterion besides an extra term $\sum \gamma$ which however does not influence the solution since it is independent of the control. The problem including the crossterm can be transformed into the standard form by introducing a new control variable u' [1].

$$\underline{u}'_k = R^{-1} M^T\ \underline{x}_k + \underline{u}_k \tag{16}$$

which transforms ((6),(7),(8)) into

$$\underline{x}_{k+1} = \Phi'_k \underline{x}_k + \Gamma_k \underline{u}'_k + \underline{v} \tag{17}$$

$$J' = E\{\int_0^{t_f} \underline{x}_k^T Q' \underline{x}_k + \underline{u}_k^T R' \underline{u}_k + \underline{x}_N^T H \underline{x}_N\} \tag{18}$$

where

$$\Phi' = \Phi - \Gamma R^{-1} M^T \tag{19}$$

$$Q' = Q - M R^{-1} M^T \tag{20}$$

The linear discrete time stochastic regulator problem ((16),(17),(18)) has a useful solution if

$Q' > 0$ and $R > 0$. It has been proved [3] that $Q>0$ implies $Q' > 0$ which garantees that the solution of the EDSRP exists.

3. Transformation of the tracking problem.

The desired movement of the orthogonal robot is assumed to be known as a function of time, i.e. the desired movement of each axes is assumed to be known as a function of time. The movement of each axes is uniquely determined by the translation of each axes, from a reference position, as a function of time. We will refer to this translation as

$$f(t) \quad 0<t<t_f \tag{21}$$

The reference trajectory $\underline{x}_0$ appearing in the tracking problem ((2),(3),(4)) includes both link translation x and link speed $\dot{x}$. The desired link speed equals df(t)/dt. If the trajectory

$$\underline{x}_0 = [\, f(t), df(t)/dt \,]^T \tag{22}$$

can be modeled as the output of a linear time invariant reference model

$$\dot{\underline{x}}_{ref} = A_{ref}\, \underline{x}_{ref} \tag{23}$$

$$\underline{x}_0 = C_{ref}\, \underline{x}_{ref} \tag{24}$$

the tracking problem ((2),(3),(4)) is to find the piecewise constant control (3) that minimizes (4) subjected to the link model (2) and the reference model ((23),(24)). Given ((23),(24)) equation (4) becomes

$$J = E[\int_0^{t_f} [[I,-C_{ref}] \begin{matrix} \underline{x} \\ \underline{x}_{ref} \end{matrix}]^T Q [I,-C_{ref}] \begin{matrix} \underline{x} \\ \underline{x}_{ref} \end{matrix} + u^T R u \, dt$$

$$+ \quad [[I,-C_{ref}] \begin{matrix} \underline{x}(t_f) \\ \underline{x}_{ref}(t_f) \end{matrix}]^T H [I,-C_{ref}] \begin{matrix} \underline{x}(t_f) \\ \underline{x}_{ref}(t_f) \end{matrix} \;]$$

If we introduce the augmented model

$$\dot{\underline{x}}_a = A_a\, \underline{x}_a + B_a\, \underline{u}_a + \underline{w}_a \tag{25}$$

where

$$A_a = \begin{bmatrix} A & 0 \\ 0 & A_{ref} \end{bmatrix} \tag{26}$$

$$B_a = \begin{bmatrix} B \\ 0 \end{bmatrix} \tag{27}$$

$$\underline{w}_a = \begin{bmatrix} \underline{w} \\ \underline{0} \end{bmatrix} \tag{28}$$

$$\underline{x}_a = \begin{bmatrix} \underline{x} \\ \underline{x}_{ref} \end{bmatrix} \tag{29}$$

we may write (25) as

$$J = E[\int_0^{t_f} \underline{x}_a^T Q' \underline{x}_a + \underline{u}^T R \underline{u} \; dt + \underline{x}_a^T(t_f)\, H' \underline{x}_a(t_f)] \tag{30}$$

with

$$Q' = \begin{bmatrix} Q & -QC_{ref} \\ -C_{ref}^T Q & C_{ref}^T Q C_{ref} \end{bmatrix} \tag{31}$$

$$H' = \begin{bmatrix} H & -HC_{ref} \\ -C_{ref}^T H & C_{ref}^T H C_{ref} \end{bmatrix} \tag{32}$$

The resulting linear stochastic regulator problem ((3),(25),....,(32)) has a useful solution if $Q'>0$, $R>0$ and $H'>0$. If in (4) the actual state $\underline{x}$ is unequal to the desired state $\underline{x}_0$ then since $Q>0$ the term $(\underline{x}-\underline{x}_0)^T Q(\underline{x}-\underline{x}_0)$ will be greater or equal to zero. If $\underline{x}$ equals $\underline{x}_0$ then this term will be zero. Since in (30) $\underline{x}^T Q' \underline{x} = (\underline{x}-\underline{x}_0)^T Q(\underline{x}-\underline{x}_0)$ this term will be greater or equal to zero for any $\underline{x}$, which simply means $Q'>0$. The same applies for H'.

4. Trajectory approximation

The remaining problem is to model the trajectory (22) as an output of a linear time invariant system ((23),(24)). This modeling determines the matrices A_{ref} and C_{ref} in the linear stochastic regulator problem with piecewise constant control ((3),(25),...,(32)). The following is well known from fourier analyses [5]

Given

$$f(t) \quad 0<t<t_f$$

then

$$f(t) = 1/2a_0 + \sum_{k=0}^{\infty} a_k \cos 2\pi k t/t_f + b_k \sin 2\pi k t/t_f \tag{33}$$

where

$$a_k = 2/t_f \int_0^{t_f} \cos(2\pi k t/t_f) f(t) dt \tag{34}$$

$$b_k = 2/t_f \int_0^{t_f} \sin(2\pi k t/t_f) f(t) dt \tag{35}$$

If we truncate the summation (33) at k=M the result is a least squares approximation of f(t). It minimizes

$$I = \int_0^{t_f} (f(t) - a(t))^2 dt \tag{36}$$

where

$$a(t) = 1/2\ A_0 + \sum_{k=0}^{M} A_k \cos(2\pi t/t_f) + B_k \sin(2\pi t/t_f) \tag{37}$$

and A ,B are the parameters to be optimized. In other words (36) is minimized choosing $A_k = a_k$ and $B_k = b_k$, $k=0,1,..,M$. The minimum value of (36) obtained in this way equals

$$I = t_f/2 \sum_{k=M+1}^{\infty} (a_k^2 + b_k^2) \tag{38}$$

Note that when f(t) is bandlimited, which it always is in practice, the approximation can be exact if M is sufficiently large. Any term

$$a_k \cos(2\pi kt/t_f) + b_k \sin(2\pi t/t_f) \tag{39}$$

is a solution of

$$\ddot{x} = -2\pi t_f^{-1} k^{-2}\ x \tag{40}$$

so can be modeled by

$$\begin{bmatrix} \dot{x}_k \\ \ddot{x}_k \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -2\pi t_f^{-1}\ k^{-2} \end{bmatrix} \begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} \tag{41}$$

where x_k equals (39) and $\dot{x}_k$ is its time derivative

So

$$\underline{x}_k = \begin{bmatrix} x_k \\ \dot{x}_k \end{bmatrix} = \begin{bmatrix} a_k \cos(2\pi t/t_f) + b_k \sin(2\pi t/t_f) \\ (2\pi/t_f)\left[a_k \sin(2\pi t/t_f) - b_k \cos(2\pi t/t_f)\right] \end{bmatrix} \tag{42}$$

which implies that the state vector $\underline{x}_k$ of (41) is completely known (42). Using (41),(42) repeatedly we can approximate

$$\underline{x}'_0 = \left[f(t) - 1/2\ a_0, df(t)/dt\right]^T \tag{43}$$

by

$$\dot{\underline{x}}_{ref} = A_{ref}\ \underline{x}_{ref} \tag{44}$$

$$\underline{x}'_0 = C_{ref}\ \underline{x}_{ref} \tag{45}$$

where

$$A_{ref} = \begin{bmatrix} \begin{matrix} 0 & 1 \\ 0 & -2\pi t_f^{-1} \end{matrix} & & & & 0 \\ & \ddots & & & \\ & & \begin{matrix} 0 & 1 \\ 0 & -2\pi t_f^{-1} k^{-2} \end{matrix} & & \\ & & & \ddots & \\ 0 & & & & \begin{matrix} 0 & 1 \\ 0 & -2\pi t_f^{-1} M^{-2} \end{matrix} \end{bmatrix} \tag{46}$$

and C_{ref} consists of M 2x2 unity matrices

$$C_{ref} = [\ I, I, \ldots\ldots.. I] \tag{47}$$

The trajectory $\underline{x}'_0$ (43) differs from the original trajectory (22) in that the desired link translation differs by an amount of 1/2 a_0. If we introduce a state transformation acting on (1)

$$\underline{x}' = \begin{bmatrix} x + 1/2\ a_0 \\ \dot{x} \end{bmatrix} \tag{48}$$

which does not affect A and B then the tracking problem ((2),(3),(4)) equals the problem where $\underline{x}'$ tracks $\underline{x}'_0$. The transformation (48) can be regarded as a change of the reference position from which the link translation is measured.

5. Control algorithm

The linear quadratic stochastic tracking problem with piecewise constant control ((2),(3),(4)) for each link of the orthogonal robot modeled by (1) can be transformed into a linear quadratic stochastic regulator problem ((3),(25),..,(32)) with time invariant system matrices which has a known solution where

$$\underline{x}_a = \begin{bmatrix} \underline{x}' \\ \underline{x}'_0 \end{bmatrix} \tag{49}$$

with $\underline{x}'$ and $\underline{x}'_0$ determined by (48),(45) A_{ref}, C_{ref} by (46,47) The solution of this regulator problem is known (chapter 2) and takes the following form

$$u(t) = u(kT_s) = F(k)\ \underline{x}_a(kT\ )\ , k=0,1,..,N-1,$$

$$kT_s < t < (k+1)T_s \tag{50}$$

where F(k) are feedback gain matrices that can be calculated in advance. According to (49) we can partition the control law (50).

$$u(t) = u(kT_s) = F'(k)\ \underline{x}'(kT_s) + F'_0(k)\ \underline{x}'_0(kT_s) \tag{51}$$

where

$$F = [\ F' \quad F_0'\ ] \tag{52}$$

Note that $\underline{x}_0'$ is a 2M dimensional vector used to approximate the trajectory and note that $F_0'\underline{x}_0'$ can be calculated in advance. The dimension of the controller equals 2M+2, where M is the number of sine and cosine functions used to approximate the trajectory. Clearly increasing the accuracy of the approximation increases the dimension of the controller. However the number of on-line calculations $F'\underline{x}'$ is not affected by this. The controllaw (50) is feasable since $\underline{x}'$, given by (48), which represents link position and speed, can be measured using an encoder and possibly a tacho generator, or can be reconstructed from these measurements using a Kalman Filter. Controllaw (50) could also be used to calculate an open loop piecewise constant control which can be used together with a seperately designed digital perturbation controller.

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