International Journal of Systems Science, 2014
Vol. 45, No. 3, 509–522, http://dx.doi.org/10.1080/00207721.2012.724102

Taylor & Francis
Taylor & Francis Group

Equivalent optimal compensation problem in the delta domain for systems with white stochastic parameters

L.G. van Willigenburg[a]* and W.L. de Koning[b]

[a]*Systems and Control Group, Wageningen University, PO Box 17, 6700 AA Wageningen, The Netherlands;* [b]*Department of Mathematics, Delft University of Technology, Delft, The Netherlands*

(*Received 14 October 2011; final version received 24 June 2012*)

The equivalent representation in the delta domain of the digital optimal compensation problem is provided and computed in this article. This problem concerns finding digital optimal full and reduced-order output feedback controllers for linear time-varying and time-invariant systems with white stochastic parameters. It can subsequently be solved in the delta domain using the strengthened optimal projection equations that we recently formulated in this domain as well. If the sampling rate becomes high, stating and solving the problem in the delta domain becomes *necessary* because the conventional discrete-time problem formulation and solution become *ill-conditioned*. In this article, by means of several numerical examples and compensator implementations, we demonstrate this phenomenon. To compute and quantify the improved performance when the sampling rate becomes high, a new delta-domain algorithm is developed. This algorithm computes the performance of *arbitrary* digital compensators for linear systems with white stochastic parameters. The principle application concerns nonconservative robust digital perturbation feedback control of nonlinear systems with high sampling rates.

Keywords: optimal output feedback; perturbation feedback; delta operator; multiplicative white noise; digital optimal control

1. Introduction

Digital *perturbation* feedback control of nonlinear systems is generally performed by optimal controllers designed using quadratic criteria and linearised dynamics around the possibly optimal trajectory to be tracked by the nonlinear system (Athans 1971). To introduce *robustness* into the perturbation feedback controller design, *white stochastic parameters* may be employed in specifying the time-varying linearised dynamics about the trajectory (Bernstein and Greeley 1986; Bernstein and Hollot 1989; Banning and de Koning 1995; Hounkpevi and Yaz 2007). Doing so, a digital optimal compensation problem of the type considered in van Willigenburg and de Koning (2000) is obtained. This problem explicitly considers the *inter-sample behaviour* of the system, and the *digital nature* of the controller. This is called 'direct digital design' (Dullerud 1996) or 'design of sampled-data control systems' (Bernstein and Hollot 1989).

To solve the digital optimal compensation problem, a transformation to discrete-time must be performed first (van Willigenburg and de Koning 2000). Subsequently, the problem can be solved in discrete-time (de Koning and van Willigenburg 1998). This is represented by the horizontal level A in Figure 1. As the sampling rate becomes high, the transformation as well as the resulting equivalent discrete-time compensation problem become *ill-conditioned* leading to inaccurate compensators and associated decreased performance. As explained in Middleton and Goodwin (1990), Li and Gevers (1993) and Yuza, Goodwin, Feuer, and de Dona (2005), the ill-conditioning is overcome if the design is performed using the *delta operator* instead of the discrete-time shift operator. Recently, for time-varying linear system with white stochastic parameters, we reformulated discrete-time full and reduced-order compensator algorithms using the delta operator (van Willigenburg and de Koning 2010). In this article, we present the associated transformation to the delta domain of the digital compensation problem. Together these make up horizontal level B in Figure 1. This level enables *nonconservative robust* digital perturbation output feedback design for nonlinear systems *using high sampling rates*.

Apart from perturbation feedback control of nonlinear systems, linear systems with white stochastic parameters may appear naturally due to randomness of plant parameters (Wagenaar and de Koning 1988) or due to stochastic sampling (Immer, Yükselb, and Basar 2006; Sinopoli, Schenato, Franceschetti, Poolla, and Sastry 2008).

*Corresponding author. Email: gerard.vanwilligenburg@wur.nl

© 2014 Taylor & Francis

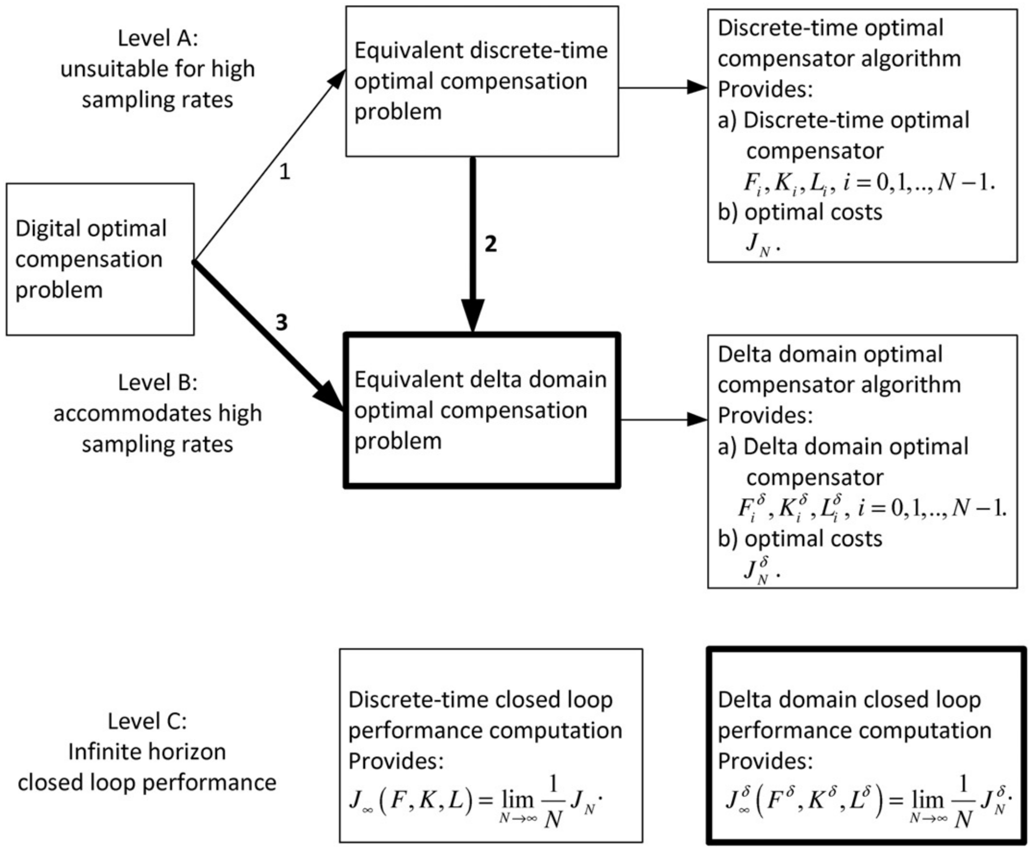


Figure 1. Discrete-time and delta-domain computations related to digital optimal and sub-optimal compensation. The new developments presented in this article are marked bold.

The transformation of digital optimal control problems with quadratic criteria into equivalent *discrete-time* problems has been specified for *time-varying* linear systems with deterministic parameters (van Willigenburg and de Koning 1992) and *white stochastic parameters* (van Willigenburg and de Koning 2000). Only if the linear system is time invariant and has deterministic parameters, transformation to equivalent problems in the delta domain is a standard exercise (Middleton and Goodwin 1990). This article presents this transformation for the general case of time-varying linear systems with white stochastic parameters.

The computational procedure proposed in this article can be briefly described as follows. As long as the sampling period is not very small, transformation into an equivalent discrete-time problem followed by a transformation to the delta domain is adopted. This is represented by arrows 1 and 2 in Figure 1. For linear time-varying systems with white stochastic parameters, the latter of these two transformations is unpublished. The transformation into an equivalent discrete-time problem is presented in Section 2. The one from discrete-time to the delta domain is presented in Section 3. As the sampling period becomes small, part of the equivalent discrete-time problem computation becomes ill-conditioned. Then, this part of the transformation must be performed avoiding the discrete-time domain as indicated by arrow 3 in Figure 1. As the sampling period becomes small, a *constant* approximation of the system dynamics over each sampling interval may be adopted. This enables and justifies the use of an algorithm that formally only applies to time-invariant linear systems with white parameters. This algorithm avoids the discrete-time domain and is presented in Section 4. It is new and combines several techniques that have been published individually. Section 5 describes the algorithm implementation for general time-varying compensation problems and arbitrary values of the sampling period. It also contains two numerical examples illustrating that the numerical inaccuracy is overcome. To further and more clearly illustrate this, a new algorithm is developed and presented in Section 6. The algorithm computes the sub-optimality of arbitrary compensators in discrete-time as well as the delta domain. It is applied to one example to quantify the performance degradation of several fixed and floating point discrete-time and delta-domain controller implementations. The outcome demonstrates the superior behaviour of delta-domain controllers when the sampling rate is high.

2. Digital optimal control problem and its discrete-time equivalent

The digital optimal compensation problem concerns a linear time-varying continuous-time system with white stochastic parameters and additive white system noise. The digital control is piecewise constant and computed from previously obtained sampled measurements (van Willigenburg and de Koning 2000). The continuous-time system, the piecewise constant control and the sampled measurements are described by

$$\mathrm{d}x(t) = \mathrm{d}A(t)x(t) + \mathrm{d}B(t)u(t) + \beta(t), \quad t_0 \le t \le t_N \quad (1)$$

$$u(t) = u(t_i),\ t \in [t_i, t_{i+1}),\ t_{i+1} > t_i, \quad i = 0, 1, \ldots, N-1 \quad (2)$$

$$y(t_i) = C_i x(t_i) + w_i, \quad i = 0, 1, \ldots, N-1 \quad (3)$$

Assumptions and details concerning the stochastic processes $\{A(t),\ t_0 \le t \le t_N\}$, $\{B(t),\ t_0 \le t \le t_N\}$, $\{\beta(t),\ t_0 \le t \le t_N\}$, $\{C_i,\ i = 0, 1, \ldots, N-1\}$ and $\{w_i,\ i = 0, 1, \ldots, N-1\}$ in Equations (1)–(3) can be found in van Willigenburg and de Koning (2000) that provides a full description of the problem. For our purpose, it suffices to consider the problem data fully specifying Equations (1)–(3) listed and explained below

$$\begin{aligned}
&\bar{x}_0 \in R^n, \quad V^{x0} \in R^{n\times n}, \quad \bar{A}(t) \in R^{n\times n},\\
&\quad \bar{B}(t) \in R^{n\times m}, \bar{C}_i \in R^{l\times n},\\
&V^{AA}(t) \in R^{n_1\times n_1}, \quad V^{AB}(t) \in R^{n_1\times n_2}, \quad V^{BA}(t) \in R^{n_2\times n_1},\\
&\quad V^{BB}(t) \in R^{n_2\times n_2},\\
&V_i^{CC} \in R^{n_4\times n_4}, \quad W_i, \quad V^{\beta}(t) \in R^{n\times n},\\
&\quad n_1 = n^2, \quad n_2 = nm, \quad n_3 = m^2, \quad n_4 = l^2.
\end{aligned} \quad (4)$$

In Equation (4), $\bar{x}_0$, V^{x0} denote the mean value and covariance matrix of the stochastic initial state $x_0 = x(t_0)$. Furthermore, $\bar{A}(t)$, $\bar{B}(t)$ and $\bar{C}_i$ denote the mean values, i.e. the first moments, of the stochastic system matrices $A(t), B(t)$ and C_i in Equations (1)–(3) where the argument $t \ge t_0$ represents continuous time and the subscript i refers to the sampling instants t_i in Equations (2) and (3). Furthermore, n, m and l in Equation (4) are the dimension of the state $x(t)$, the input $u(t)$ and the sampled output $y(t_i)$, respectively. $V^{AA}(t)$, $V^{AB}(t)$, $V^{BA}(t)$ and $V^{BB}(t)$ in Equation (4) are intensity and cross-intensity matrices of the stochastic system matrices $A(t)$ and $B(t)$. Furthermore, V_i^{CC} is the covariance matrix of the stochastic output matrix C_i. W_i are covariance matrices of the discrete-time additive white measurement noise w_i. Finally, $V^{\beta}(t)$ is the intensity matrix of the additive white system noise $\beta(t)$ in Equation (1). Observe that n_3 in Equation (4) is not formally part of the problem data. It is mentioned to remain consistent with the notation in van Willigenburg and de Koning (2000) and will be used later on.

Let E denote expectation (mean value). Then, the quadratic cost criterion to be minimised is given by

$$J = E\{x^T(t_N) Z x(t_N)\} + E\left\{\int_{t_0}^{t_N} \left[x^T(t)Q(t)x(t) + u^T(t)R(t)u(t)\right]\mathrm{d}t\right\} \quad (5)$$

$$Z \in R^{n\times n} \ge 0, Q(t) \in R^{n\times n} \ge 0, R(t) \in R^{m\times m} \ge 0 \quad (6)$$

To solve the digital optimal control problem, in van Willigenburg and de Koning (2000) this problem is converted into an equivalent discrete-time optimal control problem. This problem consists of an equivalent discrete-time linear time-varying system

$$\begin{aligned} x_{i+1} &= \Phi_i x_i + \Gamma_i u_i + v_i,\\ y_i &= C_i x_i + w_i, \quad i = 0, 1, \ldots, N-1 \end{aligned} \quad (7)$$

having white stochastic system matrices Φ_i, Γ_i and C_i and additive white system and measurement noise v_i, w_i. The data fully specifying this system are listed and explained below

$$\begin{aligned} &\bar{x}_0, \quad V_0 = V^{x_0}, \quad \bar{\Phi}_i, \quad \bar{\Gamma}_i, \quad \bar{C}_i, \quad V_i, \quad W_i,\\ &\overline{\Phi_i \otimes \Phi_i}, \quad \overline{\Phi_i \otimes \Gamma_i}, \quad \overline{\Gamma_i \otimes \Phi_i}, \quad \overline{\Gamma_i \otimes \Gamma_i}, \quad V_i^{CC} \end{aligned} \quad (8)$$

In Equation (8), $\otimes$ denotes the Kronecker product and V_i, W_i are the covariance matrices associated with the discrete-time white noise processes v_i and w_i. In Equation (8), $\bar{\Phi}_i$, $\bar{\Gamma}_i$, $\bar{C}_i$ are first moments and $\overline{\Phi_i \otimes \Phi_i}$, $\overline{\Phi_i \otimes \Gamma_i}$, $\overline{\Gamma_i \otimes \Phi_i}$ and $\overline{\Gamma_i \otimes \Gamma_i}$ are second moments of the stochastic system matrices Φ_i, Γ_i and C_i. To present the main results of this article, it is sometimes convenient to indicate the dimensions of several second moment matrices appearing in Equation (8). This is done as follows using subscripts

$$\overline{(\Phi_i \otimes \Phi_i)}_{n_1 n_1}, \quad \overline{(\Phi_i \otimes \Gamma_i)}_{n_1 n_2}, \quad \overline{(\Gamma_i \otimes \Phi_i)}_{n_2 n_1}, \quad \overline{(\Gamma_i \otimes \Gamma_i)}_{n_2 n_2} \quad (9)$$

The subscript $n_1 n_2$ in Equation (9) indicates a $n_1 \times n_2$ matrix and similarly for the others. Because Φ_i, C_i as well as Γ_i, C_i are uncorrelated, the following second moments are already specified by the data (8) (Tiedemann and de Koning 1984),

$$\overline{\Phi_i \otimes C_i} = \bar{\Phi}_i \otimes \bar{C}_i, \quad \overline{\Gamma_i \otimes C_i} = \bar{\Gamma}_i \otimes \bar{C}_i \quad (10)$$

Also

$$\overline{C_i \otimes C_i} = \bar{C}_i \otimes \bar{C}_i + V_i^{CC} \tag{11}$$

The equivalent discrete-time quadratic cost criterion

$$J_N = x_N^T Z x_N + \sum_{i=0}^{N-1} \left(x_i^T Q_i x_i + x_i^T M_i u_i + u_i^T R_i u_i + \eta_i \right) \tag{12}$$

is equal to the original cost criterion J in Equation (5) and is determined by the following data

$$Z \in R^{n \times n}, \quad Q_i \in R^{n \times n}, \quad R_i \in R^{m \times m}, \quad M_i \in R^{n \times m}, \quad \eta_i \tag{13}$$

M_i in Equation (13) determines cross products in the equivalent discrete-time quadratic cost criterion (12), whereas η_i are terms that are not influenced by the digital input but are needed to compute the costs (12). van Willigenburg and de Koning (2000) describe how to transform the data (4) and (6) of the original digital optimal control problem into the data (8) and (13) of the equivalent discrete-time optimal control problem. The part of this transformation that is sensitive to ill-conditioning needs *direct* transformation to the delta domain, indicated by arrow 3 in Figure 1. This part of the transformation is therefore stated below. Let I_n denote the identity matrix of dimension n and let the other subscripts used below indicate dimensions, as in Equation (9). Consider the following matrices that are fully specified by the problem data (4),

$$P_{n_1 n_1}(t) = \bar{A}(t) \otimes I_n + I_n \otimes \bar{A}(t) + V^{AA}(t) \tag{14}$$

$$W^1_{n_1 n_2}(t) = I_n \otimes \bar{B}(t) + V^{AB}(t) \tag{15}$$

$$W^2_{n_1 n_2}(t) = \bar{B}(t) \otimes I_n + V^{BA}(t) \tag{16}$$

$$X^1_{n_2 n_2}(t) = \bar{A}(t) \otimes I_m \tag{17}$$

$$X^2_{n_2 n_2}(t) = I_m \otimes \bar{A}(t) \tag{18}$$

$$L^1_{n_2 n_3}(t) = \bar{B}(t) \otimes I_m \tag{19}$$

$$L^2_{n_2 n_3}(t) = I_m \otimes \bar{B}(t) \tag{20}$$

$$Z_{n_1 n_3}(t) = V^{BB}(t) \tag{21}$$

Let θ denote a zero matrix. Next define

$$F(t) = \begin{bmatrix} P_{n_1 n_1}(t) & W^1_{n_1 n_2}(t) & W^2_{n_1 n_2}(t) & Z_{n_1 n_3}(t) \\ \theta_{n_2 n_1} & X^1_{n_2 n_2}(t) & \theta_{n_2 n_2} & L^1_{n_2 n_3}(t) \\ \theta_{n_2 n_1} & \theta_{n_2 n_2} & X^2_{n_2 n_2}(t) & L^2_{n_2 n_3}(t) \\ \theta_{n_3 n_1} & \theta_{n_3 n_2} & \theta_{n_3 n_2} & \theta_{n_3 n_3} \end{bmatrix} \tag{22}$$

and let $\Phi_F(t_{i+1}, t_i)$ denote the transition matrix from t_i to t_{i+1} of the homogeneous time-varying deterministic system

$$\dot{x}_F(t) = F(t) x_F(t) \tag{23}$$

Then, from Graham (1981), both the first and second moments of Φ_i and Γ_i, that are part of the equivalent discrete time problem data (8), follow from the equality

$$\begin{bmatrix} \overline{(\Phi_i \otimes \Phi_i)}_{n_1 n_1} & \overline{(\Phi_i \otimes \Gamma_i)}_{n_1 n_2} & \overline{(\Gamma_i \otimes \Phi_i)}_{n_1 n_2} & \overline{(\Gamma_i \otimes \Gamma_i)}_{n_1 n_3} \\ \theta_{n_2 n_1} & \left(\bar{\Phi}_i \otimes I_m\right)_{n_2 n_2} & \theta_{n_2 n_2} & \left(\bar{\Gamma}_i \otimes I_m\right)_{n_2 n_3} \\ \theta_{n_2 n_1} & \theta_{n_2 n_2} & \left(I_m \otimes \bar{\Phi}_i\right)_{n_2 n_2} & \left(I_m \otimes \bar{\Gamma}_i\right)_{n_2 n_3} \\ \theta_{n_3 n_1} & \theta_{n_3 n_2} & \theta_{n_3 n_2} & \theta_{n_3 n_3} \end{bmatrix} = \Phi_F(t_{i+1}, t_i) \tag{24}$$

Given $F(t)$ in Equation (22), specified by Equations (14)–(21) determined by the problem data (4), the numerical computation of $\Phi_F(t_{i+1}, t_i)$ in Equation (24) is fully treated in van Willigenburg and de Koning (2000).

3. Transformation from discrete-time to the delta domain

The transformation from discrete-time to the delta domain, presented in this section, is new. We first focus on the part of the transformation that concerns the white stochastic system matrices. To transform the problem data from discrete-time to the delta domain, first consider the delta operator δ applied to the state x_i of the equivalent discrete-time system (7),

$$\delta x_i = \frac{(x_{i+1} - x_i)}{T}, \quad T > 0 \tag{25}$$

where $T > 0$ is the fixed delta parameter (Middleton and Goodwin 1990). In the case of digital control problems, this parameter is generally associated with the sampling intervals

$$T = t_{i+1} - t_i, \quad i = 0, 1, \ldots, N-1 \tag{26}$$

Equation (26) describes periodic sampling and is used to simplify the presentation. As we will argue, all our results also apply to nonperiodic sampling.

After the transformation (25), the equivalent discrete-time system is represented in the delta domain by

$$\delta x_i = \Phi_i^\delta x_i + \Gamma_i^\delta u_i + v_i^\delta, \quad y_i = C_i^\delta x_i + w_i^\delta, \quad i = 0, 1, \ldots, N-1 \tag{27}$$

where Φ_i^δ, Γ_i^δ and C_i^δ are white stochastic system matrices and v_i^δ and w_i^δ additive white noise representations in the delta domain having associated covariance matrices V_i^δ and W_i^δ. Using Equation (25), the following relations are established that determine the transformation into the delta domain of the equivalent discrete-time system

$$\bar{\Phi}_i^\delta = \frac{(\bar{\Phi}_i - I_n)}{T}, \quad \bar{\Gamma}_i^\delta = \frac{\bar{\Gamma}_i}{T}, \quad \bar{C}_i^\delta = \bar{C}_i,$$
$$V_i^{CC\delta} = \frac{V_i^{CC}}{T^2}, \quad W_i^\delta = W_i \tag{28}$$

$$\overline{\Phi_i^\delta \otimes \Phi_i^\delta} = \frac{\left(\overline{\Phi_i \otimes \Phi_i} - \bar{\Phi}_i \otimes I_n - I_n \otimes \bar{\Phi}_i + I_n \otimes I_n\right)}{T^2} \tag{29}$$

$$\overline{\Phi_i^\delta \otimes \Gamma_i^\delta} = \frac{\left(\overline{\Phi_i \otimes \Gamma_i} - I_n \otimes \bar{\Gamma}_i\right)}{T^2} \tag{30}$$

$$\overline{\Phi_i^\delta \otimes C_i^\delta} = \frac{\left(\overline{\Phi_i \otimes C_i} - I_n \otimes \bar{C}_i\right)}{T} \tag{31}$$

$$\overline{\Gamma_i^\delta \otimes \Gamma_i^\delta} = \frac{\overline{\Gamma_i \otimes \Gamma_i}}{T^2} \tag{32}$$

$$\overline{C_i^\delta \otimes C_i^\delta} = \overline{C_i \otimes C_i} \tag{33}$$

Finally, the discrete-time cost criterion J_N in Equation (12) transforms into

$$J_N^\delta = x_N^T Z^\delta x_N + \sum_{i=0}^{N-1} x_i^T Q_i^\delta x_i + x_i^T M_i^\delta u_i + u_i^T R_i^\delta u_i + \eta_i^\delta \tag{34}$$

where $J_N^\delta = J_N$. Therefore

$$Z^\delta = Z, \quad Q_i = Q_i^\delta, \quad M_i = M_i^\delta, \quad R_i = R_i^\delta, \quad \eta_i^\delta = \eta_i \tag{35}$$

Although Equations (28)–(33) and (35) represent the problem data in the delta domain, the solution of the equivalent optimal control problem in the delta domain contains the following matrices (van Willigenburg and de Koning 2010),

$$\bar{\Phi}_i^\delta, \quad \bar{\Gamma}_i^\delta, \quad \bar{C}_i^\delta, \quad T\overline{\Phi_i^\delta \otimes \Phi_i^\delta}, \quad T\overline{\Gamma_i^\delta \otimes \Gamma_i^\delta},$$
$$T\overline{C_i^\delta \otimes C_i^\delta}, \quad T\overline{\Phi_i^\delta \otimes \Gamma_i^\delta}, \quad T\overline{\Phi_i^\delta \otimes C_i^\delta},$$
$$TV_i^{CC^\delta}, \quad TW_i^\delta, \frac{1}{T}R_i^\delta, \quad \frac{1}{T}Q_i^\delta, \quad \frac{1}{T}M_i^\delta, \quad \frac{1}{T}\eta_i^\delta \tag{36}$$

Therefore, transformation to the matrices (36) is desired to solve the equivalent optimal control problem in the delta domain. The transformation to Equation (36) is fully determined by Equations (29)–(33) multiplied by T, by Equation (35) with all the matrices except for Z^δ and Z divided by T, and by Equation (28), when we multiply the equalities involving $V_i^{CC^\delta}$ and W_i^δ by T.

4. Transformation from discrete-time to the delta domain as $T \downarrow 0$

As to the consecutive transformations, first into discrete-time and next into the delta domain, numerical problems arise as $T \downarrow 0$ if we apply the association (26). In that case, as $T \downarrow 0$, $t_{i+1} \downarrow t_i$, and the deterministic discrete-time transition matrix Φ_F in Equation (24) tends to the identity matrix. As discrete-time transition matrices tend to the identity matrix, discrete-time representations and computations become extremely sensitive to rounding errors (Middleton and Goodwin 1990).

Suppose our linear system with white stochastic parameters has time-invariant first and second moments over $[t_{i+1}, t_i)$. Then, F in Equation (22) is time invariant and using the association (26),

$$\Phi_F(t_{i+1}, t_i) = \mathrm{e}^{FT} \tag{37}$$

$$\Phi_F^\delta(t_{i+1}, t_i) = \left(\Phi_F(t_{i+1}, t_i) - I_{n_1+2n_2+n_3}\right)/T = \left(\mathrm{e}^{FT} - I_{n_1+2n_2+n_3}\right)/T \tag{38}$$

$$\lim_{T\downarrow 0} \Phi_F^\delta(t_{i+1}, t_i) = \lim_{T\downarrow 0}\left(\mathrm{e}^{FT} - I_{n_1+2n_2+n_3}\right)/T = F \tag{39}$$

Numerical computation of the right-hand side of Equation (38) reveals the sensitivity to rounding errors as $T \downarrow 0$, because it diverges from F instead of converging to it as it should according to Equation (39). This problem is overcome using an alternative computation of $\mathrm{e}^{FT} - I_{n_1+2n_2+n_3}$

$$\mathrm{e}^{FT} - I_{n_1+2n_2+n_3} = FE_{12}, \quad \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} = \mathrm{e}^{\begin{bmatrix} F & I_{n_1+2n_2+n_3} \\ \theta & \theta \end{bmatrix} T} \tag{40}$$

In Equation (40), the components of both block matrices are all real square and have identical dimensions. For all real square matrices F, numerical computations of Φ_F^δ in Equation (39), using $\frac{FE_{12}}{T}$ instead of $\frac{(\mathrm{e}^{FT} - I_{n_1+2n_2+n_3})}{T}$, do tend to F as $T \downarrow 0$ because they do not become sensitive to rounding errors. As $T \downarrow 0$ in Equation (40), $E_{12} \to TI$ both theoretically and numerically. Then, $FE_{12}/T \to F$ and the limit (39) is also obtained numerically. This is illustrated by the examples in the next section. For more information on the computation (40), we refer to van Loan (1978) and Middleton and Goodwin (1990).

Using the computation (40) circumvents the ill-conditioning of our consecutive transformation into discrete-time and the delta domain as $T \downarrow 0$. It numerically restores the property (39). But Equation (40) only applies to situations where F is time-invariant. According to Equations (14)–(22) and (26), this happens if the linear system has time-invariant statistics over each sampling interval $[t_i, t_{i+1})$, $i = 0, 1, \ldots, N-1$. Fortunately, as $T \downarrow 0$, this is approximately the case because $t_{i+1} \downarrow t_i$ due to the association (26).

5. The composite transformation and its numerical implementation

We are now in a position to state the transformation and its numerical implementation. Because transformation of the statistics associated with the equivalent discrete-time system matrices Φ_i, Γ_i may suffer from ill-conditioning, as explained in Section 4, their transformation is described separately in this section. Apart from this, transformation of the problem data to discrete-time is fully described in van Willigenburg and de Koning (2000). Part of this computation is summarised in Section 2. Next transformation to the delta domain, which is new, is described in Section 3. In this section, we finally consider the transformation of the statistics associated with the equivalent discrete-time system matrices Φ_i and Γ_i.

Starting from Equations (14)–(23), determined by the original problem data (4), we compute $\Phi_F(t_{i+1}, t_i)$ in Equation (24) by means of numerical integration, as in van Willigenburg and de Koning (2000). Over each sampling interval $[t_i, t_{i+1})$, this numerical integration is performed with a fixed time step Δt using a piecewise constant approximation of $F(t)$,

$$F(t) = F(t_i + k\Delta t), \quad t_i + k\Delta t \le t \le t_i + (k+1)\Delta t,$$
$$k = 0, 1, \ldots, K-1, \quad K = (t_{i+1} - t_i)/\Delta t, \tag{41}$$

According to van Willigenburg and de Koning (2000), the time step Δt in Equation (40) is selected such that (1) it is sufficiently small to guarantee sufficient accuracy and (2) it results in a positive integer number K. As long as the sampling rate does not become very high, $K > 1$ is obtained in Equation (40) to achieve sufficient accuracy. In this case, the ill-conditioning in discrete-time does not occur and we next compute the transformation from discrete-time to the delta-domain data (36), as described at the end of Section 3. This provides $\Phi_F^\delta(t_{i+1}, t_i)$ when $K > 1$.

As the sampling rate does become high, $K = 1$ will provide sufficient accuracy. According to Equation (40), $K = 1$ implies using a *constant* approximation of $F(t)$ over $[t_i, t_{i+1})$. In that case, the computation of Equation (24) may be performed according to Equation (37). Doing so, Equations (38)–(40) apply. In the case $K = 1$, to transform problem data associated with Φ_i, Γ_i to the delta domain, Equation (38) is used. Computation of Equation (38) is performed using Equation (40) to prevent ill-conditioning in discrete-time. This provides $\Phi_F^\delta(t_{i+1}, t_i)$ when $K = 1$.

From Equations (37), (38) and (24) observe that

$$\Phi_F^\delta(t_{i+1}, t_i) = \frac{1}{T}\left(\Phi_F(t_{i+1}, t_i) - I_{n_1+2n_2+n_3}\right) \tag{42}$$

Introduce the following notation

$$\Phi_F^\delta(t_{i+1}, t_i) = \begin{bmatrix} \Phi_F^{\delta^{11}} & \Phi_F^{\delta^{12}} & \Phi_F^{\delta^{13}} & \Phi_F^{\delta^{14}} \\ \theta & \Phi_F^{\delta^{22}} & \theta & \Phi_F^{\delta^{24}} \\ \theta & \theta & \Phi_F^{\delta^{33}} & \Phi_F^{\delta^{34}} \\ \theta & \theta & \theta & \theta \end{bmatrix} \tag{43}$$

where the sub-matrices in Equation (43) have the same dimensions as those in Equation (24), used in Equation (42). Then, from Equations (24), (42), (43) and (28)–(30) observe that

$$\bar{\Phi}_i^\delta = \Phi_F^{\delta^{33}}(1:n, 1:n) \tag{44}$$

$$\bar{\Gamma}_i^\delta = \Phi_F^{\delta^{34}}(1:n, 1:m) \tag{45}$$

$$T\overline{\Phi_i^\delta \otimes \Phi_i^\delta} = \Phi_F^{\delta^{11}} - I \otimes \bar{\Phi}_i^\delta - \bar{\Phi}_i^\delta \otimes I \tag{46}$$

$$T\overline{\Phi_i^\delta \otimes \Gamma_i^\delta} = \Phi_F^{\delta^{12}} - I \otimes \bar{\Gamma}_i^\delta \tag{47}$$

$$T\overline{\Gamma_i^\delta \otimes \Gamma_i^\delta} = \Phi_F^{\delta^{14}} \tag{48}$$

where $\Phi_F^{\delta^{33}}(1:n, 1:n)$ in Equation (44) denotes the first n rows and first n columns of $\Phi_F^{\delta^{33}}$. Similarly, $\Phi_F^{\delta^{34}}(1:n, 1:m)$ in Equation (45) denotes the first n rows and first m columns of $\Phi_F^{\delta^{34}}$. According to Equations (44)–(48), Φ_F^δ provides the delta-domain problem data in Equation (36) associated with the equivalent discrete-time system matrices Φ_i, Γ_i. When performed in the manner described in this section, the computation of Φ_F^δ does not suffer from ill-conditioning in discrete-time.

6. Principal application, examples of ill-conditioning and performance improvement of delta-domain controller computations and implementations

6.1. *Principal application and example selection*

Although 40 years old, the optimal control system design methodology proposed by Athans (1971) is still very attractive when the goal is to design a control system that is nearly optimal while the system is nonlinear and the cost function other than quadratic. The design of the control system takes place at two levels. At the top level, an optimal control problem is solved off-line, assuming the nonlinear system to be deterministic. In addition to the nonlinear nature of the system, at this top level many types of constraints and many types of cost functions can be handled making this approach highly versatile.

Because the optimal control is open loop, and the nonlinear system considered deterministic, at the second level, an output *perturbation* feedback controller (compensator) is designed. Because the compensator operates on the *perturbation* level *linearised models and quadratic criteria are appropriate* as is very well explained by Athans (1971). This is fortunate since the associated optimal compensators require only a very small number of on-line computations, as is required by feedback in many applications, even though computational power has increased significantly over the last 40 years. This motivated our developments over the years of theory and algorithms for digital optimal full and reduced-order compensator design for linear systems with deterministic and white stochastic parameters. The development of these algorithms in the delta domain is justified because: (1) they unify controller designs in continuous and discrete-time and (2) they *prevent ill-conditioning* if the sampling rate becomes high.

There is one important assumption underlying Athans approach: the errors should remain sufficiently small. This requires sufficiently accurate systems modelling which in turn justifies application of optimal control. Industrial practice often considers accurate systems modelling expensive and cumbersome. Although much inside and understanding is gained from it, accurate systems modelling is not easily achieved. To relieve the demand for accurate modelling, the compensator instead of optimal may be designed to be more robust. Using *white stochastic parameters* to represent the linearised dynamics at the perturbation level, offers a way to design a compensator that is robust on the one hand but nonconservative on the other (Bernstein and Greeley 1986). This motivated our delta-domain development in this article and in van Willigenburg and de Koning (2010) of digital compensators for linear systems with white stochastic parameters.

When developing and researching algorithms, we always use numerical examples for verification and to demonstrate, initially to ourselves, feasibility and key properties. If these are the only goals, as in this article, we preferably keep the examples small and simple. Therefore, in this article, we have deliberately avoided industrial examples, since these often introduce complications unrelated to the algorithm development. These may obscure our goals and moreover they require much more room to describe. This does not mean to say that we consider industrial applications unimportant. On the contrary: control is an *applied* science and considering industrial cases to us is a *major*, *next* research step.

6.2. *Examples of ill-conditioning*

To illustrate the prevention of ill-conditioning and to simplify the presentation, the next examples concern two infinite-horizon time-invariant compensation examples taken from van Willigenburg and de Koning (2010). The infinite-horizon time-invariant nature implies that all matrices in Equations (4) and (5), representing the cost function and the statistics of the continuous-time system, are assumed time invariant. Furthermore, the following quadratic cost function applies

$$J_\infty = \lim_{N\to\infty} \frac{1}{N} J_N \tag{49}$$

where J_N is the equivalent discrete-time cost function (12) with

$$\begin{aligned} R_i = R, \quad Q_i = Q, \quad M_i = M, \\ \eta_i = \eta, \quad i = 0, 1, \ldots, N-1 \end{aligned} \tag{50}$$

due to the time-invariant nature of Equations (4) and (5).

Example 1: Consider a digital optimal compensation problem with data (4) and (6) equal to those of Example 1 in van Willigenburg and de Koning (2010), except for $W_i = 10^{-4}\mathrm{diag}\begin{pmatrix}1 & 1\end{pmatrix}$. Choose $\beta = 0.3$.

Example 2: Consider a digital optimal compensation problem with data (4) and (6) equal to those of Example 3 in van Willigenburg and de Koning (2010), except for $W_i = 10^{-4}\mathrm{diag}\begin{pmatrix}1 & 1\end{pmatrix}$. Choose $\beta = 2$.

From van Willigenburg and de Koning (2010), observe that V^{AA}, V^{BB} and V^{CC} in Example 1 are nonzero, while Example 2 is a singular infinite-horizon time-invariant continuous-time compensation problem that is highly difficult to solve numerically. It could

Table 1. Matrix norms that should theoretically be of the order $T = 10^{-12}$.

Example	Equation (40)	$\lVert\bar{\Phi}_i^\delta - \bar{A}\rVert$	$\lVert\bar{\Gamma}_i^\delta - \bar{B}\rVert$	$\lVert T\overline{\Phi_i^\delta \otimes \Phi_i^\delta} - V^{AA}\rVert$	$\lVert T\overline{\Gamma_i^\delta \otimes \Gamma_i^\delta} - V^{BB}\rVert$	$\lVert T\overline{\Phi_i^\delta \otimes \Gamma_i^\delta} - V^{AB}\rVert$
1	Used	4.5828E − 12	2.0529E − 12	1.8514E − 11	2.4696E − 12	4.6474E − 12
1	Not used	2.2225E − 05	2.0529E − 12	9.8253E − 05	2.4696E − 12	4.6474E − 12
2	Used	7.9992E − 14	1.9984E − 13	1.6009E − 13	2.6006E − 12	4.0012E − 13
2	Not used	9.7667E − 05	1.9984E − 13	2.2204E − 04	2.6006E − 12	4.0006E − 13

only be solved by approximating it by consecutive equivalent optimal control problems in the delta domain. These equivalent optimal control problems in the delta domain were actually computed using the algorithm presented in the previous section. By taking ever smaller values for T, down to 10^{-6}, while using the solution of the previous problem to initialise the next, a numerical solution was obtained in van Willigenburg and de Koning (2010).

A first demonstration of how the ill-conditioning is prevented is given by Table 1. This table states matrix norms which should theoretically be of the order T (van Willigenburg and de Koning 2010) which we selected to be 10^{-12}. The norms concern the equivalent problem data in the delta domain associated with Φ_i and Γ_i of Example 1 and Example 2. The norms were computed using Matlab with a machine constant of 2.22×10^{-16}. Because theoretically the norms are of the order $T = 10^{-12}$, Table 1 clearly reveals that errors occur in $\bar{\Phi}_i^\delta$ and $T\overline{\Phi_i^\delta \otimes \Phi_i^\delta}$ when we do not adjust the computation according to Equation (40), to prevent ill-conditioning. The magnitude of the errors in Table 1, however, may still be acceptable for most controller computations. In the next section, using a similar example, we will further investigate the numerical ill-conditioning and the way it is prevented by computations in the delta domain.

Finally, consider again Equation (26). This equation described periodic sampling. Because the linear system dynamics and statistics, as well as the quadratic criterion matrices, are time-varying in general, the computations described in this article are different for each sampling interval $[t_i, t_{i+1})$, $i = 0, 1, \ldots, N-1$, even if the sampling is periodic. Making *separate* computations for each sampling interval $[t_i, t_{i+1})$, $i = 0, 1, \ldots, N-1$, allows us to change T in Equation (26) for every such computation and interval. Doing so, we effectively change Equation (26) into

$$T_i = t_{i+1} - t_i, \quad i = 0, 1, \ldots, N-1 \tag{51}$$

describing nonperiodic sampling. Stated differently, when restricting the application of the delta-domain representation in this article to each individual sampling interval $[t_i, t_{i+1})$, $i = 0, 1, \ldots, N-1$, we may select the delta parameter differently for each individual sampling interval, as described by Equation (51). Doing so, the results of this article apply to nonperiodic sampling as well.

6.3. *Performance improvement of delta-domain controller computations and implementations*

Example 3: Consider a digital optimal compensation problem with data (4) and (6) equal to those of Example 1 in van Willigenburg and de Koning (2010), except for $R(t) = 10^{-4}\text{diag}(1 \quad 1)$ and $W_i = 10^{-4}\text{diag}(1 \quad 1)$. Choose $\beta = 0.03$.

The *computation* of optimal compensators, of the type considered in this article, is generally performed off-line, on advanced computers, e.g. running Matlab. Matlab uses a highly accurate, double precision floating point representation of real numbers. Despite this fact, the discrete-time optimal compensator algorithm, when executed in Matlab, becomes *ill-conditioned* when the sampling time becomes very small. This is illustrated by Figure 2 that displays the minimum costs computed for Example 3 using both the discrete-time and delta-domain optimal compensation algorithm. The algorithms are represented by the top right boxes in Figure 1. Only when performed in the delta domain, represented by the horizontal level B in Figure 1, the optimal costs converge nicely to the one obtained in continuous-time, as they should.

The *implementation* of compensators often takes place on platforms with less accurate representations of real numbers, such as micro-controllers, which are sometimes still fixed point. For several fixed and floating point representations of real numbers, in this section we will quantify the optimal controller performance degradation, when using discrete-time and delta-domain fixed and floating point controller implementations. This will demonstrate the superiority of delta-domain controller implementations if the sampling rate is high. Computation of controller performance degradation requires an algorithm, executed in Matlab, that computes the performance of a *closed-loop system* that consists of an *arbitrary* compensator applied to our linear system with

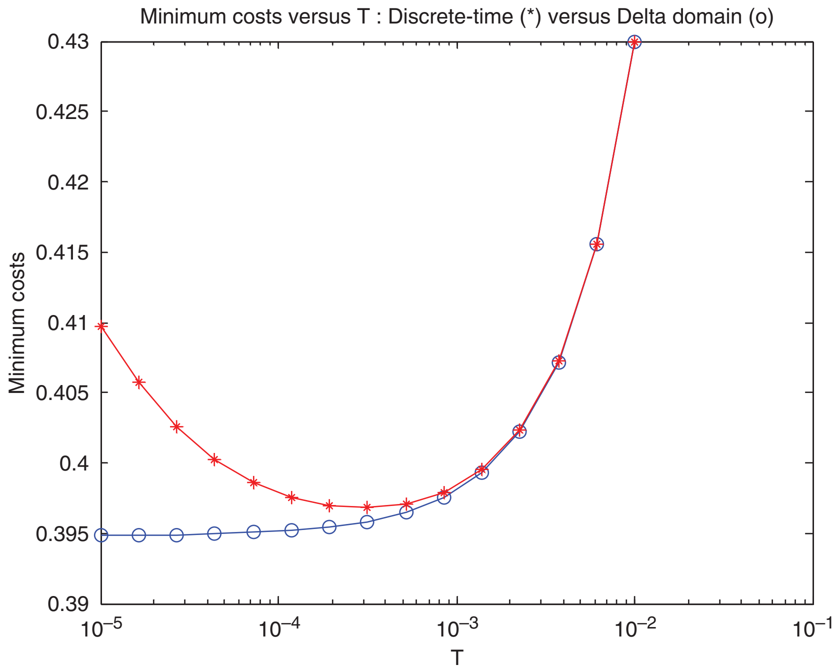


Figure 2. Minimum costs (Example 3) obtained from the discrete-time and delta-domain optimal compensation algorithm.

stochastic parameters. This should preferably be a delta-domain algorithm because the investigated control systems are by definition close to being ill-conditioned, due to the high sampling rate.

To simplify the presentation, the investigation and algorithm presented in this section are again restricted to the infinite horizon time-invariant case. The algorithms for the finite horizon time-varying are easily obtained from them. To obtain the algorithm, first consider the closed-loop control system in discrete-time

$$\begin{bmatrix} x_{i+1} \\ \hat{x}_{i+1} \end{bmatrix} = \begin{bmatrix} \Phi_i & -\Gamma_i L \\ KC_i & F \end{bmatrix} \begin{bmatrix} x_i \\ \hat{x}_i \end{bmatrix} + \begin{bmatrix} v_i \\ Kw_i \end{bmatrix}, \quad i = 0, 1, \ldots, N-1 \tag{52}$$

The matrices Φ_i, Γ_i and C_i and vectors v_i and w_i are those of the equivalent discrete-time system (7). Furthermore, x_i represents the state of this system and $\hat{x}_i$ the full- or reduced-order compensator state (de Koning 1992). Their first and second moments are time invariant because the matrices in Equation (4) are time invariant. Define

$$x'_i = \begin{bmatrix} x_i \\ \hat{x}_i \end{bmatrix}, \quad v'_i = \begin{bmatrix} v_i \\ Kw_i \end{bmatrix}, \quad \Phi'_i = \begin{bmatrix} \Phi_i & -\Gamma_i L \\ KC_i & F \end{bmatrix}, \quad V' = \begin{bmatrix} V & \theta \\ \theta & KWK^T \end{bmatrix} \tag{53}$$

Then, the closed-loop system (52) may be written as

$$x'_{i+1} = \Phi'_i x'_i + v'_i \tag{54}$$

Let $P'_i = \overline{x'_i x'^T_i}$, the second moment of the closed-loop system. Then, from Equation (54),

$$P'_{i+1} = \overline{\Phi'_i P'_i \Phi'^T_i} + V' \tag{55}$$

When the closed-loop system is mean square stable (ms-stable),

$$P' = \lim_{i\to\infty} P'_i \tag{56}$$

exists, $P' \geq 0$ and P' is the unique solution of

$$P' = \overline{\Phi' P' \Phi'^T} + V' \tag{57}$$

where the time index i of Φ is deleted because $\overline{\Phi'_i P' \Phi'^T_i}$ is independent of i in the time-invariant case (de Koning 1992). Equations (55) and (57) are generalised Lyapunov equations. The quadratic criterion (49) has a finite value given by de Koning (1992) and van Willigenburg and de Koning (2004),

$$J_\infty(F, K, L) = \lim_{N\to\infty} \frac{1}{N} J_N = tr(Q'P'), \quad Q' = \begin{bmatrix} Q & -ML \\ -L^T M^T & L^T RL \end{bmatrix} \tag{58}$$

To compute the criterion value (58) we need to compute the limit (56), using Equation (55). This constitutes the discrete-time algorithm to compute the closed-loop system performance. This algorithm is represented by the left box at the horizontal level C in Figure 1. The closed-loop performance computed by this algorithm is denoted by $J_\infty(F, K, L)$.

Because instead of a discrete-time algorithm, we prefer a delta-domain algorithm, below we derive the delta-domain equivalent of Equation (55). First observe from de Koning (1992) that

$$\overline{\Phi' P_i \Phi'^T} = st^{-1}\left(\overline{\Phi' \otimes \Phi'} st(P_i)\right) \tag{59}$$

where st denotes the stack operator. Furthermore

$$\begin{aligned}\overline{\Phi' \otimes \Phi'} &= \overline{(T\Phi'^{\delta} + I) \otimes (T\Phi'^{\delta} + I)} \\ &= T^2\overline{\Phi'^{\delta} \otimes \Phi'^{\delta}} + T\overline{\Phi'^{\delta}} \otimes I \\ &\quad + TI \otimes \overline{\Phi'^{\delta}} + I \otimes I\end{aligned} \tag{60}$$

$$\begin{aligned}\overline{\Phi' \otimes \Phi'}^{\delta} &= T^{-1}\left(\overline{\Phi' \otimes \Phi'} - I \otimes I\right) \\ &= T\overline{\Phi'^{\delta} \otimes \Phi'^{\delta}} + \overline{\Phi'^{\delta}} \otimes I + I \otimes \overline{\Phi'^{\delta}}\end{aligned} \tag{61}$$

Using Equations (59)–(61) we obtain from Equation (55),

$$\begin{aligned}P_{i+1} &= \overline{\Phi' P_i \Phi'^T} + V = st^{-1}\left(\overline{\Phi' \otimes \Phi'} st(P_i)\right) + V \\ &= T\left(st^{-1}\left(\left(\overline{\Phi' \otimes \Phi'}^{\delta}\right) st(P_i)\right) + TV^{\delta}\right) + P_i.\end{aligned} \tag{62}$$

From Equation (62),

$$\frac{P_{i+1} - P_i}{T} = st^{-1}\left(\overline{\Phi' \otimes \Phi'}^{\delta} st(P_i)\right) + TV^{\delta} \tag{63}$$

which, using Equations (61) and (59), corresponds to the generalised delta-domain Lyapunov equation

$$\delta P = T\overline{\Phi'^{\delta} P \Phi'^{\delta T}} + \overline{\Phi'^{\delta}} P + P\overline{\Phi'^{\delta T}} + TV^{\delta} \tag{64}$$

To compute the limit (56), we iterate the delta-domain equivalent (62) of Equation (55),

$$P_{i+1} = T\left(st^{-1}\left(\left(\overline{\Phi' \otimes \Phi'}^{\delta}\right) st(P_i)\right) + TV^{\delta}\right) + P_i \tag{65}$$

starting from $P_i = \theta$ until convergence. As in de Koning (1992), the trace of P_i is used to detect convergence. The value of the delta parameter T formally equals the sampling interval. Since we need to find the steady-state solution of Equation (65), the objective of the algorithm is to quickly find the value of P_i, that sets to zero the term $(st^{-1}((\overline{\Phi' \otimes \Phi'}^{\delta})st(P_i)) + TV^{\delta})$ in Equation (65). To achieve this, the first value of T in Equation (65) may actually be changed into a different positive value, to promote faster convergence, if possible. Associated with this observation, in the continuous-time case, i.e. in the limit $T \downarrow 0$, TV^{δ} becomes constant while the first appearance of T in Equation (65) may be interpreted as the step size of Euler numerical integration (van Willigenburg and de Koning 2010). The performance of the closed-loop system computed by this delta-domain algorithm, represented by the right block of the horizontal level C in Figure 1, is denoted by $J^{\delta}_{\infty}(F^{\delta}, K^{\delta}, L^{\delta})$.

To mimic the implementation of discrete-time and delta-domain controllers in fixed or floating point processors, the discrete-time controller matrices and the delta-domain controller matrices were rounded in Matlab to an associated number of digits. Next, the rounded discrete-time controller matrices were converted to the delta domain. Then, for both types of controllers, using the delta-domain algorithm described in this section, the associated closed-loop performance, represented by $J^{\delta}_{\infty}(F^{\delta}, K^{\delta}, L^{\delta})$ in Figure 1, was computed in Matlab. Figures 3–6 represent the results. In Figures 4 and 6, for small T, several values are unrecorded because the closed-loop system was no longer ms-stable implying infinite costs since the limit (56) tends to infinity. The figures clearly illustrate the significant loss of performance, and sometimes even loss of ms-stability, associated with the implemented discrete-time controllers, when the sampling rate becomes very small. The figures also reveal the superior performance and behaviour of the implemented delta-domain controllers. Note that the closed-loop computations do not incorporate the rounding of measurement and control values, that also takes place in digital controllers. Therefore, the computed loss of performance is conservative.

7. Conclusions

Equivalent optimal control problem formulations in discrete-time are generally used for digital optimal control system design. They explicitly consider the continuous-time performance (inter-sample behaviour) as well as the sampling phenomena. As the sampling rate becomes very high, equivalent discrete-time optimal control problem formulations become ill-conditioned numerically. As a result, their numerical solution and the associated compensator implementation become inaccurate and inefficient. For time-varying and time-invariant compensation problems, involving systems with white stochastic parameters, this problem is circumvented in this article by computing equivalent optimal control problems in the delta domain. For sampling intervals larger than the time-step required for numerical integration, transformation to discrete-time followed by a transformation to the delta domain is used. The latter transformation is presented for the first time in this article. For sampling intervals larger than the time-step required for numerical integration, ill-conditioning in discrete-time does not occur and time-varying compensation problem data can be handled. For sampling intervals less or equal to the time-step required for numerical integration, the problem data may be approximated over each sampling interval by time-invariant data. If the

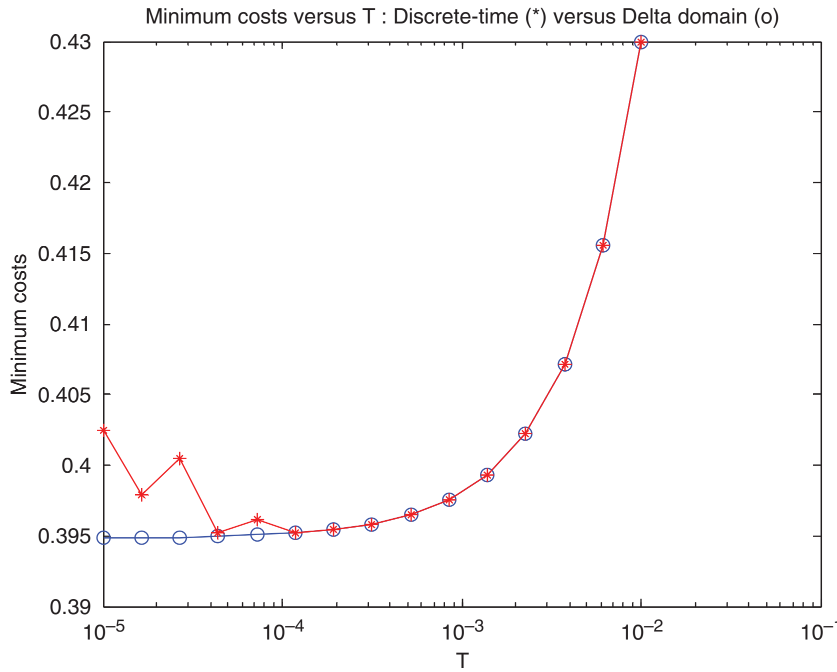


Figure 3. Minimum costs (Example 3) for optimal discrete-time and delta-domain floating point control implementations with a three-digit mantissa.

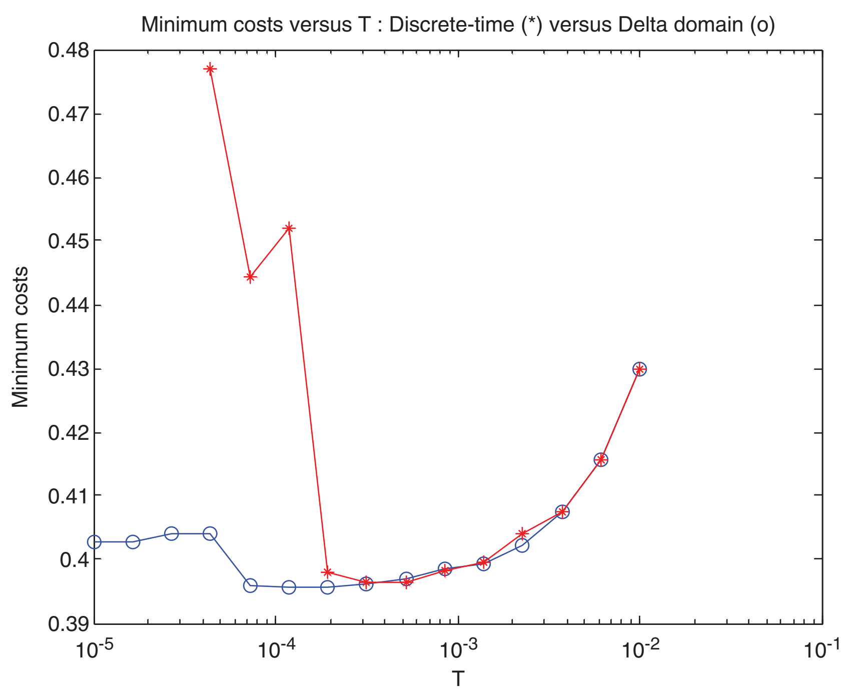


Figure 4. Minimum costs (Example 3) for optimal discrete-time and delta-domain floating point controller implementations with a two-digit mantissa.

problem data are time invariant, an alternative transformation to the delta domain, presented for the first time in this article, is adopted that does not suffer from ill-conditioning in discrete-time. The principle application of the results presented in this article, and in van Willigenburg and de Koning (2010), concerns the robust compensation of nonlinear systems along optimal control and state trajectories (Athans 1971).

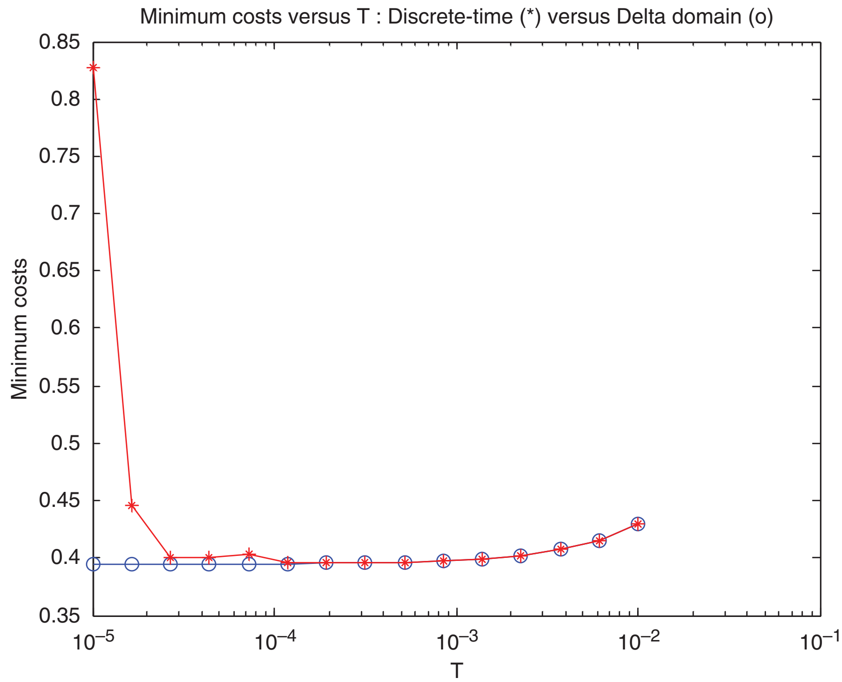


Figure 5. Minimum costs (Example 3) for optimal discrete-time and delta-domain fixed point controller implementations with three digits.

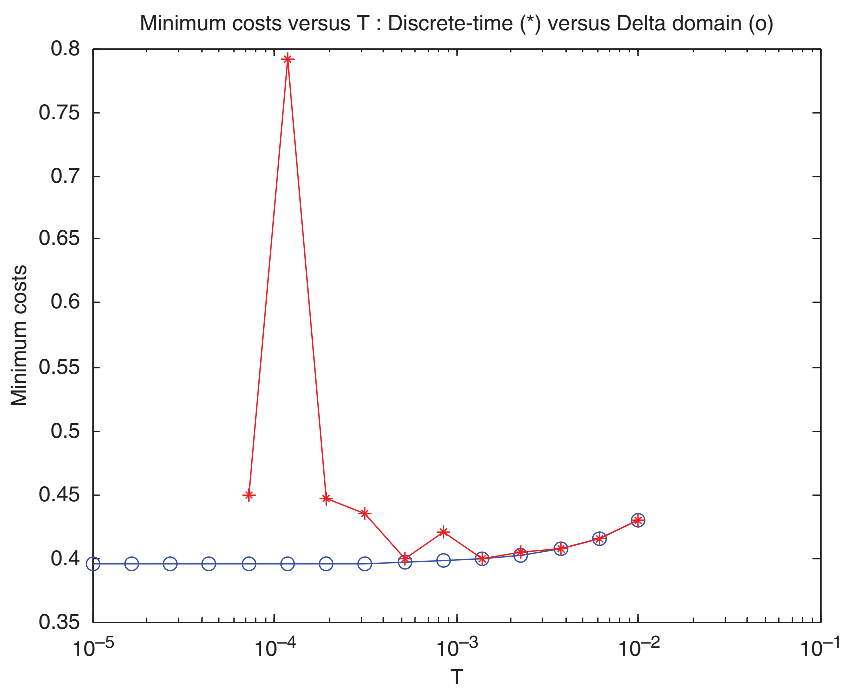
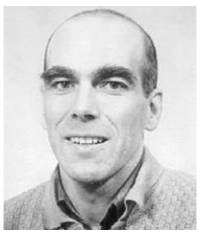


Figure 6. Minimum costs (Example 3) for optimal discrete-time and delta-domain fixed point controller implementations with two digits.

Numerical examples were used to illustrate and quantify the performance degradation of discrete-time controllers when the sampling time becomes very small. The same examples showed that this degradation is largely prevented by working in the delta domain.

Throughout this article we assumed synchronous sampling, meaning that all control variables and measurements are updated simultaneously at the sampling instants. Allowing for asynchronous sampling requires the introduction of systems models and compensators having time-varying dimensions (van Willigenburg and de Koning 2001, 2008). To accommodate for this in the delta domain requires an interesting generalisation of the delta operator that we are currently investigating.

Notes on contributors



L.G. van Willigenburg was born in Leiden, the Netherlands, in 1958. He received his MSc in electrical engineering and his PhD degree in control engineering from Delft University of Technology, the Netherlands, in 1983 and 1991, respectively. From 1986 to 1991, he was a research engineer at the process dynamics and control group in the department of applied physics. Since 1991, he is an assistant professor of the systems and control group at Wageningen University. His professional research interests include digital optimal control, reduced-order control, model predictive control and adaptive dual control. The application areas range from indoor climate control (storage rooms, greenhouses, stables and buildings), robot control, automatic guidance, to process control (fermentation) and the control of economic and biological systems (plants). The modelling of fundamental physics (thermodynamics and quantum mechanics) has become a private research interest.

W.L. de Koning was born in Leiden, the Netherlands, in 1944. He received his MSc in electrical engineering and his PhD degree in control engineering at Delft University of Technology, the Netherlands, in 1975 and 1984, respectively. From 1969 to 1975, he was a research engineer of power electronics and control at the department of electrical engineering. From 1987 to 2006, he was an associate professor of mathematical system theory at the department of mathematical system theory in the department of technical mathematics and informatics. He has also held a visiting position at the Florida Institute of Technology, Melbourne. His research interests include control of distributed parameter systems, robust control, adaptive control, reduced-order control and digital control. The main application areas are the process industry and mechatronics. Although he retired in 2006, he continues some of his research on a personal basis.

References

Athans, M. (1971), 'The Role and Use of The Stochastic Linear-Quadratic-Gaussian Problem in Control System Design', *IEEE Transactions on Automatic Control*, AC-16, 529–552.

Banning, R., and de Koning, W.L. (1995), 'Robust Control Using White Parameters to Model The System Uncertainty', in *Proceedings of the European Control Conference*, 5–8 September, Rome, Italy, pp. 2504–2508.

Bernstein, D.S., and Greeley, S.W. (1986), 'Robust Controller Synthesis Using The Maximum Entropy Design Equations', *IEEE Transactions on Automatic Control*, AC-31, 362–364.

Bernstein, D.S., and Hollot, C.V. (1989), 'Robust Stability for Sampled-data Control Systems', *Systems and Control Letters*, 13, 217–226.

de Koning, W.L. (1992), 'Compensatability and Optimal Compensation of Systems with White Parameters', *IEEE Transactions on Automatic Control*, 37, 579–588.

de Koning, W.L., and van Willigenburg, L.G. (1998), 'Numerical Algorithms and Issues concerning the Discrete-time Optimal Projection Equations for Systems with White Parameters', in *Proceedings of the UKACC International Conference on Control '98*, 1–4 September, University of Swansea, UK, Vol. 2, pp. 1605–1610.

Dullerud, G.E. (1996), *Control of Uncertain Sampled-data Systems*, Boston, MA: Birkhauser.

Graham, A. (1981), *Kronecker Products and Matrix Calculus with Applications*, Chichester, West Sussex: Ellis Horwood.

Hounkpevi, F.O., and Yaz, E.E. (2007), 'Robust Minimum Variance Linear Estimators for Multiple Sensors with Different Failure Rates', *Automatica*, 43, 1274–1280.

Immer, O.C., Yükselb, S., and Basar, T. (2006), 'Optimal Control of LTI Systems over Unreliable Communication Links', *Automatica*, 24, 1429–1439.

Li, G., and Gevers, M. (1993), 'Comparative Study of Finite Wordlength Effects in Shift and Delta Operator Parameterizations', *IEEE Transactions on Automatic Control*, 38, 803–807.

Middleton, R.H., and Goodwin, G.C. (1990), *Digital Control and Estimation: A Unified Approach*, Englewood Cliffs, NJ: Prentice-Hall.

Sinopoli, O.B., Schenato, L., Franceschetti, M., Poolla, K., and Sastry, S. (2008), 'Optimal Linear LQG Control over Lossy Networks Without Packet Acknowledgment', *Asian Journal of Control*, 10, 3–13.

Tiedemann, A.R., and de Koning, W.L. (1984), 'The Equivalent Discrete-time Optimal Control Problem for Continuous Time Systems with Stochastic Parameters', *International Journal of Control*, 40, 449–466.

van Loan, C. (1978), 'Computing Integrals Involving the Matrix Exponential', *IEEE Transactions on Automatic Control*, AC-23, 395–404.

van Willigenburg, L.G., and de Koning, W.L. (1992), 'The Digital Optimal Regulator and Tracker for Stochastic Time-varying Systems', *International Journal of Systems Science*, 12, 2309–2322.

van Willigenburg, L.G., and de Koning, W.L. (2000), 'The Equivalent Discrete-time Optimal Control Problem for Time-varying Continuous-time Systems with White Stochastic Parameters', *International Journal of System Science*, 31, 479–487.

van Willigenburg, L.G., and de Koning, W.L. (2001), 'Synthesis of Digital Optimal Reduced-order

Compensators for Asynchronously Sampled Systems', *International Journal of System Science*, 32, 825–835.

van Willigenburg, L.G., and de Koning, W.L. (2004), 'UDU Factored Discrete-time Lyapunov Recursions Solve Optimal Reduced-order LQG Problems', *European Journal of Control*, 10, 588–601.

van Willigenburg, L.G., and de Koning, W.L. (2008), 'Linear Systems Theory Revisited', *Automatica*, 44, 1669–1683.

van Willigenburg, L.G., and de Koning, W.L. (2010), 'Compensatability and Optimal Compensation of Systems with White Parameters in The Delta Domain', *International Journal of Control*, 83, 2546–2563.

Wagenaar, T.J.A., and de Koning, W.L. (1988), 'Stability and Stabilizability of Chemical Reactors Modeled with Stochastic Parameters', *International Journal of Control*, 49, 33–44.

Yuza, J., Goodwin, G., Feuer, A., and de Doná, J. (2005), 'Control of Constrained Linear Systems Using Fast Sampling Rates', *Systems and Control Letters*, 54, 981–990.