Meeting you, Djaeni and Cindy

@ Diponegoro University
The mathematical model consists of (first-order) differential equations recognisable by presence of rates of change $dx/dt$.

Models containing differential equations are called dynamic models.

Dynamic models are obtained from knowledge of the system, often scientific knowledge (first-principles models / white box models).

Using scientific knowledge allows us to understand and interpret the model!

Black box models do not allow for this.
Optimal control problem: Given a mathematical dynamic model of the system find the control patterns (time-functions/trajectories/histories) that maximize (minimize) a cost function (performance measure/cost criterion/penalty function).

Problem formulation is still missing one thing ...

We also have to know “where the system starts”

We have to know the initial state : values of W(0), T(0), CO₂(0).
Summary (1)

- With optimal control:
  - All types of systems can be handled.
  - All types of control objectives can be considered.
  - All types of constraints can be handled.

- This requires:
  - A quantitative mathematical dynamic model of the system obtained by exploiting scientific knowledge.
  - A quantitative mathematical description of the control objectives.
  - A quantitative mathematical description of the constraints.

- What you get: The best control & performance!

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Summary (2)

- Measurements:
  - are *only* required to realize feedback to counteract model and other types of *uncertainty*.
Mathematical Modelling & Optimal Control

Optimal Control System

- Control Computer
  - Optimal control
    - $u^*(t)$
  - Output feedback
    - $u^*(t), x^*(t), y^*(t)$
      - $y^*(t)$
      - Output perturbations
        - $\Delta y(t)$
        - On-line
    - Actual output
      - $y(t)$
- Actual control
  - $u(t)$
  - Control corrections
    - $\Delta u(t)$
    - Off-line
  - System
    - $u(t)$

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Mathematical Modelling & Optimal Control

State space models, simulation and steady states

- Learn how to obtain a state-space model systematically
- Use state-space models to perform simulations
- Steady state computation: Simple control problem
- Continuation lecture on black board
Summary (1)

- We learned how to obtain a state-space model systematically
  - Make sketch/drawing with mathematical symbols
  - From (mass/energy/..) balance equations obtain differential equations (use of scientific knowledge)
  - 1) Find states $x$ (those variables showing time derivatives)
  - 2) Find parameters $p$ (all constants)
  - 3) Find inputs $u$ (variables that remain)
  - From this obtain $\frac{dx}{dt} = f(x,u,p)$
  - 4) Find outputs $y$ (measured variables/variables of interest)
  - From this obtain $y = g(x,u,p)$

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Using it on a computer we can perform simulations:
- From the input trajectories (time functions) we calculate the state response (state trajectories = system behaviour) and the output response (output trajectories)
- Simulations are needed to compute optimal controls

Steady state computation: Simple control problem
- Compute controls to keep states constant
- Do small deviations from steady state tend to zero? If yes steady state stable, otherwise not
- Check this by computing eigenvalues of \( \frac{\partial f}{\partial x} \bigg|_{x=x^{ss}, u=u^{ss}} \)
- Properties of \( f, x^{ss}, u^{ss} \) determine this