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A MODEL OF FISH GROWTH

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ABSTRACT

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A fish model based upon mass balances is set up. Several of the parameters have been determined by experiment. The remaining parameters are based upon literature values and it was not necessary to find any parameter by calibration. The model was validated with a completely acceptable result.

The model includes equations giving the fish growth, the production of ammonia and organic matter, the oxygen consumption, and the influence of the temperature.

The model can be used for management of fish farms and as a submodel for a total aquatic system.

INTRODUCTION

A model of fish growth has, besides scientific interest, also a possible application in two connexions:

(1) As submodel in an ecological total model of an aquatic environment.

(2) For management of fish farms.

The model set up is considered in the first place to be used for management purposes, but also some indications of its use as submodel in a total model of an aquatic environment will be given.

THE MODEL

The model set up is based upon the principles of mass balance, which have already been applied in this context by Pütter (1920).

As shown in Fig. 1 the feed, F is divided into

(1) feed which is biochemically oxidized for metabolism (RE),

(2) feed which is not digested ($NDF(F - NUF)$),

(3) feed which is used as energy for assimilation (ALG),

(4) surplus feed which cannot be utilized (NUF), and

(5) growth = increase of biomass (dW/dt).

The equations of the model are shown in Table I, and Table II gives a list

TABLE I

Equation

A

(1) $\frac{dW}{dt} = \text{PDMFFR}(F - NUF)(1 - NDF)(1 - ALC) - RE$

(2) $\text{PDMFFR} = \text{PMFEED/PMFISH}$

(3) $NUF = 0, \quad \text{when } W^{EA} \geqslant 2\ WFC \cdot F$

(4) $NUF = F - \frac{W^{EA}}{2\ WFC}, \quad \text{when } W^{EA} \leqslant 2\ WFC \cdot F$

(5) $NDF = \frac{WFC \cdot F}{W^{EA}}, \quad \text{when } NUF = 0$

(6) $NDF = 0.5, \quad \text{when } NUF > 0$

(7) $RE = RC \cdot W^{ER}$

(8) $FCO = \frac{F}{dW}$

B

(9) $\frac{dBOD}{dt} = (1 - \text{OOFC})\ (F - NUF)\ (NDF + NUF)$

C

(10) $\frac{dNP}{dt} = \frac{AEC}{6.15}\left(\frac{W}{1}\right)^{EM} + \text{OOFC} \cdot PPFEED\left(\frac{(F - NUF)(NDF + NUF)}{100 \times 6.15}\right)$

(11) $PPFEED = \text{PPFISH} \cdot \text{PDMFFR}\left(\frac{dW}{(F - NUF)(1 - NDF)}\right) + \left(\frac{AEC \cdot 100 \cdot W^{EM}(1 - NDF)}{F - NUF}\right)$

D

(12) $\frac{dOC}{dt} = \text{OOR} \cdot \text{OOFC}((F - NUF)(NDF + ALC - NDF \cdot ALC) + \text{OOR} \cdot \text{RMR} \cdot RE)$

E

(13) $F = FF(0.0077\ W^{EA} + 0.0022\ W^{ER})$

(14) $F = FF(0.009\ W^{EA})$

F

(15) $AEC = 0.038\ TEMP^{1.142} \times 10^{-3}$

(16) $ALC = 0.021\ TEMP^{0.703}$

(17) $RC = 0.776\ TEMP^{0.821} \times 10^{-3}$

(18) $WFC = 4.158 \times 10^{2}\ TEMP^{-0.807}$

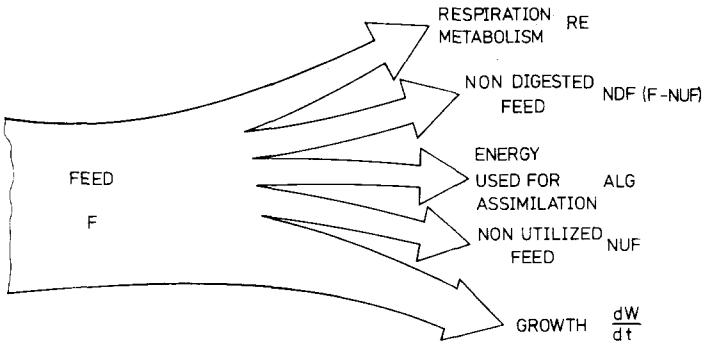

Fig. 1. The principle of the model of fish growth is shown. The feed is used for respiration, is non-digested, is used for the assimilation processes, is not utilized and is used for growth. Notice that *ALG* is equal to $(F - NUF)$ $(1 - NDF)$ *ALC* and only $(F - NUF)$ $(1 - NDF)$ $(1 - ALC)$ is available for respiration and growth.

TABLE II

Symbols explanation

Symbol	Variable (V) or parameter (P)	Explanation	Unit
AEC	V *	Ammonia excretion coefficient	1/24 h
ALC	V *	Assimilation loss coefficient	—
ALG	V	Assimilation loss = PDMFFR $(F - NUF)$ $(1 - NDF)$ *ALG*	kg/24 h
BOD	V	Production of BOD	kg/24 h
EA	P	Appetite exponent	—
EM	P	Metabolism exponent	—
ER	P	Respiration exponent	—
F	V **	Feed	—
FF	V ***	Feed factor	—
FCO	V	Feeding coefficient	—
NDF	V	Non digested feed coefficient	—
NP	V	Ammonia production	kg/24 h
NUF	V	Non utilized feed	kg/24 h
OC	V	Oxygen consumption	kg/24 h
OOFC	P	Fraction of organic matter oxidized in the fish tank	—
OOR	P	Ratio oxygen to organic matter	—
PDMFFR	P	PMFEED/PMFISH	—
PMFEED	P	% dry matter in feed	—
PMFISH	P	% dry matter in fish	—
PPFEED	V	% protein in feed	—
PPFISH	P	% protein in fish	—
RC	V *	Respiration coefficient	1/24 h
RE	V	Respiration loss	kg/24 h
RMR	P	Respiration—metabolism ratio	—
W	V	Weight	
WFC	V *	Wasted feed coefficient	—

* Temperature dependent.
** A function of times.
*** Chosen separately in each case.

of the symbols used. Equation A (1) is the mathematical expression of Fig. 1. PDMFFR is a constant converting dry matter in the feed to dry matter in the fish. The part of the feed utilized is $(F - NUF)$ of which only $(F - NUF)$ $(1 - NDF)$ $(1 - ALC)$ is available for metabolism RE and growth dW/dt. The non-digested feed (NDF) is given in equation A (8). NDF is proportional to the ratio, the feed to the appetite. The fact that NDF increases when this ratio increases is a general ecological principle (see e.g. Odum, 1971).

The form it has in this case is in accordance with Ursin (1972). The respiration is generally proportional to the surface (Odum, 1971), which corresponds to equation A (6), where ER is 0.7—0.85 (Ursin, 1967; Halver, 1970). From experiments it has been shown (see later) that when NDF is as high as 0.5 (50% of the feed is not digested) a further increase of the feed will not be utilized. This is expressed in equations A (2) and A (3). For management purposes it is of importance to calculate FCO — the feeding coefficient. This is done by means of equation A (7). B (9) gives the production of organic matter (detritus). By multiplication with OOFC, the BOD production is obtained.

The excretion of ammonium is a metabolic contribution (Halver, 1970) and when the model is used for management of fish farms the part of the organic matter oxidized in the fish tank must be added. The right protein concentration of the feed can be found by application of equation C (11).

Equation D (12) calculates the oxygen production necessary for the fish growth — again an equation which is mainly of interest for fish farming. The same is the case with E (13) and E (14) which give two different relations

TABLE III

Parameters

Symbol	Definition	Value used	The value used is based upon
EA	Appetite exponent	0.65	Ursin (1973) Halver (1970)
EM	Metabolism exponent	0.72	Halver (1970)
ER	Respiration exponent	0.80	Ursin (1973) Ursin (1967)
OOFC	Fraction of organic matter oxidized in the fish tank	0.1	Measurement mass balance
OOR	Ratio oxygen to organic matter	1.08	Analysis
PMFEED	% dry matter in feed	93	Analysis
PMFISH	% dry matter in fish	28	Analysis
PPFISH	% protein in fish	18	Analysis
RMR	Respiration—metabolism ratio	0.24	Measurement mass balance
PDMFFR	PMFEED/PMFISH	$\frac{93}{28}$	Analysis

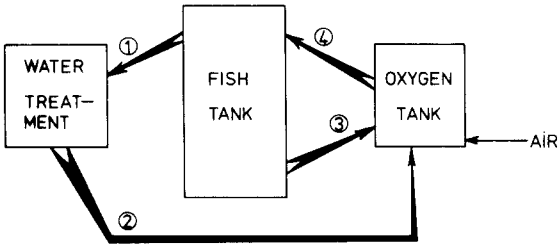

Fig. 2. Flow diagram for the experimental plant. The water quality is maintained at 90—95% oxygen saturation by means of aeration, and the NH_4^+-N concentration was kept at 1.5 mg/l or below by means of a water treatment unit, using selective ion exchangers.

between the weight of the fish and the feeding. These two equations express two different fits to feeding tables generally used (see Ewos, 1975). The feeding factor *FF* gives the possibilities of varying the feeding level relative to these equations.

Four parameters depend upon the temperature. These relationships are given in F (15) and F (18). The relationships set up are based upon Speece (1973) (a review publication giving *F*, NH_3 excretion, respiration and BOD_5 production as functions of the temperature). A regression analysis of these data has given the equations F (15)—F (18).

PARAMETER VALUES

Two series of experiments were carried out using an 11 m^3 fish tank. A flowsheet of the plant is shown in Fig. 2.

The first series was carried out during July and August 1974 at a temperature of 16.0 ± 0.3°C with approximately 750 trout with an average weight of about 275 g. The purpose was to find OOFC and RMR using mass balances. The oxygen consumption and the ammonia production was measured over a period of 24 h with feeding and 24 h without feeding, having an interval of 24 h between the two periods with 12 h reduced feeding and 12 h without feeding. The oxygen consumption was calculated from the oxygen concentration of the water at the inlet and outlet (see Fig. 2 points 4 and 3 respectively) multiplied with the water flow (it was found that the oxygen uptake from the atmosphere gave only an insignificant correction).

The ammonium production was determined by means of the following equation:

$$NH_4\text{-}N_p = (NH_4\text{-}N_1 - NH_4\text{-}N_2)Q + (NH_4\text{-}N_{T2} - NH_4\text{-}N_{T1})V$$

where:

Q = waterflow

NH_4-N_p = ammonium production (24 h)
NH_4-N_1 = ammonium concentration at outlet (Fig. 2, point 1)
NH_4-N_2 = ammonium concentration at Fig. 2, point 2
NH_4-N_{T1} = ammonium concentration in the fish tank at start of the experiment
NH_4-N_{T2} = ammonium concentration in the fish tank at the end of the experiment
V = volume of fish tank

The oxygen consumption without feeding is equal to OOR · RMR · *RE* (see Table I). The ammonium production without feeding is correspondingly equal to $(AEC/6.15)W^{EM}$ (see Table I) and the experiment can be used to test this part of the model.

The oxygen consumption when feeding, can be used to test equation D (12), while the increase in ammonium production when feeding compared with the experiment without feeding, is equal to $OOFC \cdot PPFEED\ (F - NUF)(NDF + NUF)/100 \times 6.15$ and *OOFC* can be found. These experiments were repeated twice, and the results are shown in Table IV. This table gives the percentage deviation between theoretical and experimental values. A second series of experiments was carried out to confirm the relations A (2) and A (3). This series consists of five different periods, where the feeding rate was maintained at the same value relative to W^{EA}/WFC, calculated from day to day on the basis of equation A (1).

The growth rate was measured by weighing about 10% of the fish before and after the experimental period. The results are summarized in Table V. *NUF* has been calculated on the basis of the average growth rate using equation A (1), and compared with *NUF* found on the basis of A (2) and A (3). As seen, the agreement between the two *NUF* values is quite acceptable. This must not be taken as a final confirmation of the equations but it can be concluded that the equations give at least an approximate picture of the *NUF* value. Further experiments must be carried out. The very best would be a

TABLE IV

Determination of OOFC and RMR on the basis of experiments

Experiment	Feeding	Ammonium		Oxygen	
		Deviation (%)	OOFC	Deviation (%)	RMR
1	Normal	—	0.12	−8	—
1	Without	7	—	—	0.24
2	Normal	—	0.08	4	—
2	Without	−1	—	—	0.24

TABLE V

Determination of *NUF* on the basis of experiments

Start of period	March 17th, 1974	April 20th, 1974	August 21st, 1974	April 24th, 1975	March 12th, 1975
Length of period (days)	14	14	10	14	10
Number of fish	1840	1786	724	1880	1861
Average weight of fish at start (g)	75	139	301	109	151
Temperature (°C)	16.1	15.8	17.2	15.8	15.5
Average growth (kg) per 24 h per fish	1.8×10^{-3}	2.7×10^{-3}	4.1×10^{-3}	2.2×10^{-3}	2.7×10^{-3}
Average kg feed per fish per 24 h (A)	1.7×10^{-3}	2.7×10^{-3}	7.6×10^{-3}	2.6×10^{-3}	3.6×10^{-3}
Average W^{EA}/WFC (B)	4.4×10^{-3}	6.7×10^{-3}	11.1×10^{-3}	5.7×10^{-3}	6.9×10^{-3}
(B)/(A)	2.59	2.48	1.46	2.19	1.92
NUF calculated from equation A (1)	0	0	2.1	0	0.12
NUF calculated from equations A (2)—A (4)	0	0	2.05	0	0.15

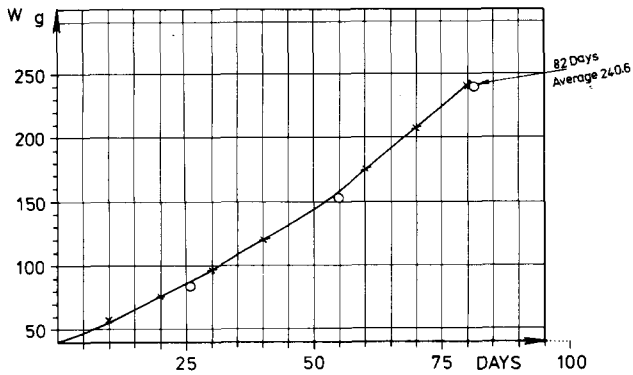

Fig. 3. The fish weight is shown as a function of the time. + indicates values found by means of the model, o measured values. Average weight when the experiment was terminated: 240.6 g.

direct measurement of the *NUF* value under different feeding rates and temperatures.

During both series of experiments an analysis of OOR, PMFEED, PMFISH and PPFISH was carried out (see Table III).

VALIDATION OF THE MODEL

The model was tested for a growth period in which about 1800 fish were reared from 49 to 240 g. The weight of the fish was determined three times during this period by weighing 200 fish. *FF* was chosen to be 0.9 and feeding was in accordance with equation E (14). The oxygen saturation was maintained at 90—95%, the NH_4^+-N concentration at 1.5 mg/l or below, and the temperature at 16 ± 0.3°C. This should guarantee excellent growth conditions (see Spotte, 1970).

The theoretical values calculated on the basis of the model are compared with the measured values in Fig. 3. Table VI gives the total production of

TABLE VI

Comparison theoretical values/measured data of the total production of BOD_5 and NH_3 per fish during the growing period

	BOD_5 (g)	NH_3 (g)
Theoretical value (model)	54	3.3
Measured data (mass balance)	57	2.9

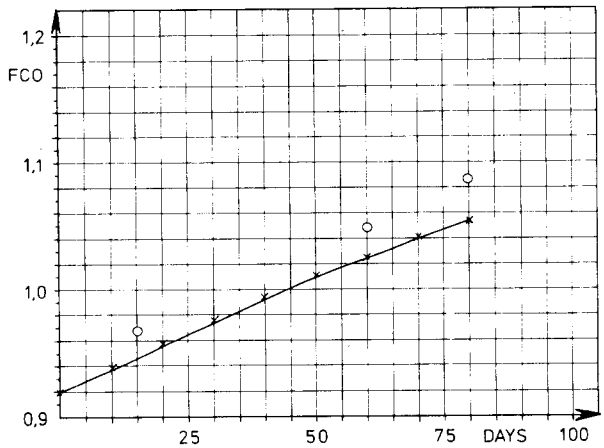

Fig. 4. The feeding coefficient is shown as a function of the time. + indicates values found on basis of the model, and ○ gives measured data.

BOD_5 and NH_3 calculated using the model and compared with the values found by means of an analytical program measuring the daily BOD_5 and NH_4-N concentration before and after the water treatment unit and in the tank during the experimental period.

Fig. 4 gives $FCO = f$ (time) in accordance with the model. The fish were weighed, as mentioned above, three times during the experimental period. The corresponding FCO value was calculated and is indicated in Fig. 4. As seen, the agreement between measured and calculated values is acceptable.

DISCUSSION

The experiments referred to have shown it is possible to validate a model of fish growth and to determine experimentally several essential parameter values.

Closed loop fish farming seems to be an important way of working out more causal fish models, as it is possible to maintain the feed relative to the appetite as well as the environmental factors almost constant.

On the basis of the results, it can be stated that the model gives completely acceptable results at 16° C, but the model was not validated at other temperatures. Furthermore, it is necessary to determine the parameter as a function of the temperature, although the equations F (15)—F (18) are based upon observation (Speece, 1973). At a later stage, similar experiments to those described in this paper are to be carried out at other temperatures to validate the part of the model describing the influence of this important forcing function. It is also necessary to validate the model for other sizes of fish (1 to 5000 g). Other species might have other parameter values, which should also be determined.

For management purposes, the model can be expanded with economic equations containing the cost figures of such factors as the feed, maintenance, interest and depreciation, to be able to optimize a fish farm.

When the fish model is used as a submodel in a model of an aquatic environment, certain changes of the equations in Table I must be carried out. Different size classes of fish must be treated separately. In this context F will be the feed available for the considered size class, this means zooplankton, small fish etc., and can be expressed by means of:

$$F = \frac{\text{feed concentration (kg biomass/m}^3)}{\text{fish (number/m}^3)}$$

All the equations A (1)—A (18) can be applied directly. The total biomass of the ecosystem can be found by multiplying W by the number of fish.

Equation B (9) discribes the production of detritus per fish, by setting OOFC = 0. Equation C (10) gives the ammonium production per fish if OOFC = 0. Equations C (11), D (12), E (14) and E (15) are not used at all in this context, while F (15)—F (18) must be included.

Hatching can be included in the model by transferring a part of the biomass at a certain time to the variable "biomass of eggs".

The mortality of the fish has not been included in the model as the number of dead fish was limited. Although the numbers were avaiable from long-term closed loop fish farming, they could not be used in the natural environment, as parasites will strongly influence the mortality there. Mortality must especially be included for hatching, and here laboratory values might give a fairly good approach.

CONCLUSION

A fish model based on mass balances has been set up and several parameters of the model were determined experimentally, using closed loop fish farming. The model is shown in Table I, equations A (1)—F (18). It has been validated applying the parameters found in separate experiments. It is remarkable that all parameters used (see Table III) are based upon literature values or own experiments. None of the parameters were found by calibration.

On the basis of the results, it can be concluded that the model gives a completely acceptable description of fish growth in the interval 50 to 240 g at 16° C. The influence of the temperature is included in the model in equations F (15)—F (18), based upon observations reviewed by Speece (1973) but there still seems to be a need to validate this part of the model as well as the growth throughout the total weight interval (e.g. 1 to 5000 g).

The results indicate that by means of closed loop fish farming it is possible to work out more causal fish models, which can also be used as submodels in models of total aquatic systems.

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