Ecological Modelling, 2 (1976) 147—165

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A EUTROPHICATION MODEL FOR A LAKE

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(Received August 29th, 1975)

ABSTRACT

Jørgensen, S.E., 1976. A eutrophication model for a lake. *Ecol. Modelling*, 2:147—165.

Different lake models were tested against measured values. It was found that a model based upon phytoplankton population dynamics gave a better description than a model based upon Monod's kinetics. The calibration of the model showed that it was necessary to use a recently developed submodel for sediment—water nutrient exchange. Furthermore, the work showed that it was essential to include in the model three trophic levels, denitrification, a time dependent nitrogen input and Steele's expression for the grazing rate.

A prediction for different waste-water treatment alternatives was worked out.

INTRODUCTION

During the past decade, much concern has been shown because of the eutrophication of Scandinavian lakes. In Sweden chemical precipitation of waste water is widely used to reduce eutrophication, while in Denmark discussion has arisen as to whether or not the reduction of nitrogen in waste water has a better effect than phosphorus.

The eutropication model set up attempts to indicate not only which nutrient one should necessarily remove, but also the concentration required in the effluent to produce improved lake conditions.

The model has been tested on Glumsø Lake, which was chosen for the following four reasons:

(1) The lake has an excessive algal growth. The transparency during the summer months is only 15—20 cm. Any improvement had to be significant.

(2) The morphology is simple. The surface area is 266,000 m^2, the average depth is 1.8 m and the maximum depth 2.4 m. The volume is 420,000 m^3. No thermocline appears.

(3) The hydraulic budget indicates that a quick response should be expected. Approximately 10^6 m^3 flows through the lake per year, giving a retention

time of about 5 months. Slightly more than 25% — 270,000 m^3 per year — is mechanically—biologically treated waste water having a high concentration of nutrients (4.5 mg P/l, 24 mg N/l).

(4) The waste water treatment is expected to be improved in 1976—1977 in accordance with the results of the model. In that case the predictions can be verified.

THE MODEL

Two different models were tested — one based upon Monod's kinetics and the other based upon some new concepts of phytoplankton population dynamics. In the first model the growth rate is determined by the external substrate concentration, while in the second model the growth rate is determined by the internal substrate concentration.

The equations, on which the first model is based, are shown in Table I. Table II gives the values of parameters, the symbols used, and how the values were found.

Table III shows the variables, their definitions and units. Table IV gives the forcing functions, their definitions, units and how they were found. CSMP was used.

The model set up differs from previous lake models, see e.g. Chen (1970), Chen and Orlob (1972, 1975), Larsen et al. (1972), Schofield and Krutchkoff (1974) and Scavia and Park (1976).

The most marked points which according to the calibration were important to include, were:

(1) The exchange of nutrient between sediment and water is described by a submodel set up on the basis of Kamp-Nielsen (1974, 1975), Jørgensen et al. (1975) and Jacobsen et al. (1975). A simple constant or first order process has previously been used for this exchange reaction. Describing the influence of this reaction on the nutrient balance of the lake is quite insufficient (see Jørgensen et al., 1973). However, the submodel takes into consideration the decrease in the exchange rate as the nutrient concentration in the sediment decreases, and distinguishes between exchangeable and unexchangeable nutrients. As measurements show that exchangeable phosphorus and phosphorous concentration in the interstitial water are both fluctuating within one year but constant from year to year, it has been possible to calibrate this submodel separately, which should give more certain parametric values.

(2) Three trophic levels are included — phytoplankton, zooplankton and fish. Because of the high concentration of phytoplankton, calibrating the model without all three trophic levels was impossible.

(3) Denitrification is included. The rate constant is based upon the nitrogen balance (see equation (12), Table I: KD · *NS*/365 · V).

(4) Using an exponential expression for the influence of the temperature on the rates of growth of phytoplankton, zooplankton and fish was found necessary and 16.5°C was used as the average optimum temperature.

TABLE I

Model 1

$$P4 = PS + DP + (CA + CZ)\mathrm{PP} \tag{1}$$

$$N4 = NS + DN + (CA + CZ)\mathrm{PN} \tag{2}$$

$$\mu A = \mu A_{\max} \cdot \exp\left(-2.3\left|\frac{T-16.5}{15}\right|\right) \cdot \mathrm{Light} \cdot \frac{C}{\mathrm{KC}+C} \cdot \frac{NS}{\mathrm{KN}+NS} \cdot \frac{PS}{\mathrm{KP}+PS} \tag{3}$$

$$\mu Z = \mu Z_{\max} \cdot \exp\left(-2.3\left|\frac{T-16.5}{15}\right|\right) \frac{CA-0.5}{\mathrm{KA}+CA}, \text{ for } CA > 0.5 \tag{4}$$

$$RA = RA_{\max} \cdot \exp\left(-2.3\left|\frac{T-16.5}{15}\right|\right) \tag{5}$$

$$RZ = RZ_{\max} \cdot \exp\left(-2.3\left|\frac{T-16.5}{15}\right|\right) \tag{6}$$

$$\mu F = \mu F_{\max} \cdot \exp\left(-2.3\left|\frac{T-16.5}{15}\right|\right) \frac{CZ}{\mathrm{KZ}+CZ} \tag{7}$$

$$\frac{\mathrm{d}CA}{\mathrm{d}t} = (\mu A - RA - \mathrm{SA} - \mathrm{MA})\, CA - \frac{\mu Z \cdot CZ}{\mathrm{F}} - \frac{\mathrm{F4} \cdot CA}{365 \cdot \mathrm{V}} \tag{8}$$

$$\frac{\mathrm{d}CZ}{\mathrm{d}t} = (\mu Z - RZ - \mathrm{MZ})\, CZ - \frac{\mu F \cdot \mathrm{CF}}{\mathrm{F}} - \frac{\mathrm{F4} \cdot CZ}{365 \cdot \mathrm{V}} \tag{9}$$

$$\frac{\mathrm{d}PS}{\mathrm{d}t} = \frac{\mathrm{F1} \cdot \mathrm{P1} + \mathrm{F2} \cdot \mathrm{P2} + \mathrm{F3} \cdot \mathrm{P3}}{365 \cdot \mathrm{V}} + \frac{1.2(PI - PS) - 1.7}{1000} \cdot \frac{T+273}{280} \cdot \frac{\mathrm{S}}{\mathrm{V}}$$
$$- \frac{(\mu A - RA)CA}{\mathrm{PP}} + \frac{RZ \cdot CZ}{\mathrm{PP}} - \frac{\mathrm{F4} \cdot PS}{365 \cdot \mathrm{V}} + \mathrm{K4} \cdot DP \cdot \mathrm{K6}^{(T-20)} \tag{10}$$

$$\frac{\mathrm{d}DP}{\mathrm{d}t} = \frac{\mathrm{MA} \cdot CA}{\mathrm{PP}} + \frac{\mathrm{MZ} \cdot CZ}{\mathrm{PP}} - \mathrm{SA} \cdot DP - \frac{\mathrm{F4} \cdot DP}{365 \cdot V} - \mathrm{K4} \cdot DP \cdot \mathrm{K6}^{(T-20)}$$
$$+ \left(\frac{1}{\mathrm{F}} - 1\right)\left(\frac{\mu Z \cdot CZ + \mu F \cdot \mathrm{CF}}{\mathrm{PP}}\right) \tag{11}$$

$$\frac{\mathrm{d}NS}{\mathrm{d}t} = \frac{\mathrm{F1} \cdot N1 + F2 \cdot \mathrm{N2} + F3 \cdot \mathrm{N3}}{365 \cdot \mathrm{V}} - \frac{F4 \cdot NS}{365 \cdot \mathrm{V}} - \frac{KD \cdot NS}{365 \cdot \mathrm{V}}$$
$$- \frac{(\mu A - RA)CA - RZ \cdot CZ}{\mathrm{PN}} + \mathrm{K4} \cdot DN \cdot \mathrm{K6}^{(T-20)}$$
$$+ \frac{4.0 \cdot SN + 0.08}{1000} \cdot \frac{S}{V} \cdot \exp(0.151 \cdot T) \tag{12}$$

$$\frac{dDN}{dt} = \frac{\mathrm{MA} \cdot CA}{\mathrm{PN}} + \frac{\mathrm{MZ} \cdot CZ}{\mathrm{PN}} - \mathrm{K4} \cdot DN \cdot \mathrm{K6}^{(T-20)} - \frac{\mathrm{F4} \cdot DN}{365 \cdot \mathrm{V}} - \mathrm{SA} \cdot DN$$
$$+ \left(\frac{1}{\mathrm{F}} - 1\right)\left(\frac{\mu Z \cdot CZ + \mu F \cdot \mathrm{CF}}{\mathrm{PN}}\right) \tag{13}$$

$$\frac{\mathrm{d}PE}{\mathrm{d}t} = \frac{\mathrm{SA} \cdot CA \cdot \mathrm{V} \cdot 10^3}{\mathrm{f} \cdot \mathrm{PP} \cdot \mathrm{LUL} \cdot \mathrm{S}} - \mathrm{K5} \cdot PE \cdot \mathrm{K6}^{(T-20)} + \frac{\mathrm{SA} \cdot DP \cdot \mathrm{V} \cdot 10^3}{\mathrm{f} \cdot \mathrm{LUL} \cdot \mathrm{S}} \tag{14}$$

$$\frac{\mathrm{d}PI}{\mathrm{d}t} = \frac{\mathrm{K5} \cdot PE \cdot \mathrm{K6}^{(T-20)}}{1 - \mathrm{DMU}} - \frac{1.2(PI - PS) - 1.7}{\mathrm{LUL}(1 - \mathrm{DMU})} \cdot \frac{T+273}{280} \tag{15}$$

$$\frac{\mathrm{d}SN}{\mathrm{d}t} = \frac{\mathrm{SA} \cdot CA \cdot \mathrm{V}}{\mathrm{S} \cdot \mathrm{PN} \cdot \mathrm{LUL}} - \frac{4.0 \cdot SN + 0.08}{1000 \cdot \mathrm{LUL}} \cdot \exp(0.151 \cdot T) + \frac{\mathrm{SA} \cdot DN \cdot \mathrm{V}}{\mathrm{LUL} \cdot \mathrm{S}} \tag{16}$$

$$\left.\begin{aligned} I_0 &= \mathrm{f}(t) \\ T &= \mathrm{f}(t) \\ N1 &= \mathrm{f}(t) \end{aligned}\right\} \text{ forcing function based upon actual measurements} \quad \begin{aligned} &(17) \\ &(18) \\ &(19) \end{aligned}$$

$$\mathrm{Light} = \frac{\mathrm{S}}{(\alpha + \beta \cdot CA)\mathrm{V}} \cdot \ln\left(\frac{I_0 + \mathrm{KI}}{I_0 \cdot \exp((-\alpha - \beta \cdot CA)(\mathrm{V}/\mathrm{S})) + \mathrm{KI}}\right) \tag{20}$$

TABLE II

Parameters

Symbol used	Definition	Value	Found by
PP	Phosphorous content in phytoplankton	$\frac{1}{114}$	Analysis of fraction 1–80 μ
PN	Nitrogen content in phytoplankton	$\frac{7}{114}$	Analysis of fraction 1–80 μ
μA_{max}	Maximum growth rate of phytoplankton	2.0/24 h	Calibration
KI	Michaelis constant for the light intensity	400 kcal/m^2/24 h	Gargas (1975)
KC	Michaelis constant for the carbon uptake	0.5 mg/l	Chen and Orlob (1975)
KN	Michaelis constant for the nitrogen uptake	0.2 mg/l	Chen and Orlob (1975)
KP	Michaelis constant for the phosphorous uptake	0.03 mg/l	Chen and Orlob (1975)
RA_{max}	Respiration rate of phytoplankton (maximum)	0.6/24 h	Calibration
RZ_{max}	Respiration rate of zooplankton (maximum)	0.02/24 h	Calibration
MA	Mortality of phytoplankton	0.015/24 h	Calibration
SA	Settling rate	0.04/24 h	Pb and Zn analysis of sediment (Jørgensen et al., 1975)
KA	Michaelis constant for the feeding rate of zooplankton	0.5 mg/l	Chen and Orlob (1975)
KZ	Michaelis constant for feeding rate of fish	0.1 mg/l	Calibration
μF_{max}	Maximum growth rate of fish	0.015/ 24 h	Calibration
μZ_{max}	Maximum growth rate of zooplankton	0.175/24 h	Calibration
MZ	Mortality of zooplankton	0.125/24 h	Calibration
S	Surface area of the lake	270,000 m^2	Geographic data
V	Volume of the lake	420,000 m^3	Geographic data
K4	Biodegradation rate of detritus	0.1/24 h	Calibration
F	Conversion factor phytoplankton–zooplankton biomass	0.63	Chen and Orlob (1975)
f	Ratio total phosphorus/exchangeable phosphorus in sediment	$\frac{29}{12}$	Analysis of sediment (see Jørgensen et al., 1975)
LUL	Upper unstabilized layer	100 mm	Analysis of sediment (see Jørgensen et al., 1975)
K5	Biodegradation rate of organic phosphorus in sediment	0.0018/24 h	On basis of submodel (see Jørgensen et al., 1975)
K6	Temperature coefficient for biodegradation	1.02	Chen and Orlob (1975)
KD	Constant for denitrification rate	0.002	Mass balances
DMU	Dry matter of upper layer in sediment	0.925 kg/kg	Analytical determination
α	Extinction coefficient of water	0.27/m	Chen and Orlob (1975)

TABLE II (continued)

Symbol used	Definition	Value	Found by
β	Specific extinction coefficient of phytoplankton	0.18 m^2/g	Chen and Orlob (1975)
CF	Concentration of fish	0.3 mg/l	Calibration
F1	Flow rate of streams going to the lake	530,000 m^3/year	Measurements
F2	Flow rate of waste water	270,000 m^3/year	Measurements
F3	Precipitation	170,000 m^3/year	Measurements
P1	Phosphorus in streams going to the lake	0.019 mg/l	Measurements
P2	Phosphorus in waste water	4.5 mg/l	Measurements
P3	Phosphorus in rainwater	0.0015 mg/l	Measurements
F4	Flow rate of streams going from the lake	800,000 m^3/year	Measurements
N2	Nitrogen in waste water	24 mg/l	Measuremenst
N3	Nitrogen in rainwater	0.36 mg/l	Measurements

(5) A time-dependent nitrogen input was needed as the streams contain considerably more nitrogen and at the same time have a higher flow between April and July. The variation of the nitrogen input was based upon the measured values of flow and nitrogen concentrations.

TABLE III

Variables

Symbol	Definition	Unit
P4	Total phosphorus in lake water	mg/l
PS	Soluble phosphorus in lake water	mg/l
DP	Detritus phosphorus in lake water	mg/l
CA	Phytoplankton concentration	mg/l
CZ	Zooplankton concentration	mg/l
NS	Soluble nitrogen in lake water	mg/l
μA	Growth rate of phytoplankton	$(24\ h)^{-1}$
μZ	Growth rate of zooplankton	$(24\ h)^{-1}$
RA	Respiration rate of phytoplankton	$(24\ h)^{-1}$
RZ	Respiration rate of zooplankton	$(24\ h)^{-1}$
μF	Growth rate of fish	$(24\ h)^{-1}$
PI	Phosphorus of interstitial water in sediment	mg/l
PE	Exchangeable phosphorus in upper layer of sediment	mg/l
DN	Detritus nitrogen in lake water	mg/l
N4	Total nitrogen in lake water	mg/l
I	Light intensity	kcal/m^2/24 h
SN	Nitrogen in upper layer of sediment	g/l
C	Inorganic C in water, fixed value 100 mg/l is used	mg/l

TABLE IV

Forcing functions

Symbol	Definition	Unit	Found by
I_0	Light intensity at the lake surface	kcal/m^2/24 h	Measurements
T	Temperature	°C	Measurements
$N1$	Nitrogen in streams	mg/l	Measurements

(6) Different expressions for the influence of the intensity of light on the rate of growth of phytoplankton have been tested (Schofield and Krutchkoff, 1974; Scavia and Park, 1976), including the inhibition of the rate of growth at high light intensities.

The difference between the different expressions was not significant. However, it was found to be absolutely necessary to include the influence of the phytoplankton concentration on the extinction coefficient.

(7) For the zooplankton grazing rate the expression of Steele (1974) is used. 0.5 mg/l is applied as the threshold concentration below which feeding is zero. Without the threshold concentration the concentration of phytoplankton would be too low during the period October to April.

The equations, parameters and variables of the second model, based upon phytoplankton population dynamics, are given in Tables V—VII. Equations (3), (8), (9) and (10) in Table V are parallel to the equations set up by Lehman et al. (1975) for single cells. However, the equations in Table V are based upon the phytoplankton biomass.

The growth of phytoplankton is described as a two-step process: (1) uptake of nutrients into the cells, and (2) biomass growth.

TABLE V

Model 2

$$P4 = PS + DP + \frac{PC \cdot CZ}{CA} + PC \tag{1}$$

$$N4 = NS + DN + \frac{NC \cdot CZ}{CA} + NC \tag{2}$$

$$UC = \frac{\mathrm{S}}{(\alpha + \beta \cdot CA) \cdot \mathrm{V}} \cdot \ln\left(\frac{I_0 + \mathrm{KI}}{I_0 \cdot \exp((-\alpha - \beta \cdot CA)(\mathrm{V/S})) + \mathrm{KI}}\right) \cdot \frac{CC_{\max} \cdot CA - CC}{CC_{\max} \cdot CA - CC_{\min} \cdot CA} \cdot UC_{\max} \cdot CA \cdot \frac{C}{KC + C} - RA \tag{3}$$

$$\mu Z = \mu Z_{\max} \cdot \exp\left(-2.3\left|\frac{T - 16.5}{15}\right|\right) \cdot \frac{CA - 0.5}{\mathrm{KA} + CA}, \text{ for } CA > 0.5 \tag{4}$$

$$RA = RA_{\max} \cdot CA\left(\frac{CC}{CC_{\max} \cdot CA}\right)^{0.67} \cdot \exp\left(-2.3\left|\frac{T - 16.5}{15}\right|\right) \tag{5}$$

$$\mu F = \mu F_{\max} \cdot \exp\left(-2.3\left|\frac{T - 16.5}{15}\right|\right) \frac{CZ}{CZ + \mathrm{KZ}} \tag{6}$$

TABLE V (continued)

$$RZ = RZ_{max} \cdot \exp\left(-2.3\left|\frac{T-16.5}{15}\right|\right) \tag{7}$$

$$UP = UP_{max} \cdot \frac{PC_{max} \cdot CA - PC}{PC_{max} \cdot CA - PC_{min} \cdot CA} \cdot \frac{CA \cdot PS}{PS + \mathrm{KP}} \tag{8}$$

$$UN = UN_{max} \cdot \frac{NC_{max} \cdot CA - NC}{NC_{max} \cdot CA - NC_{min} \cdot CA} \cdot \frac{CA \cdot NS}{NS + \mathrm{KN}} \tag{9}$$

$$GRW = GRW_{max} \cdot \exp\left(-2.3\left|\frac{T-16.5}{15}\right|\right) \frac{NC - NC_{min} \cdot CA}{NC}$$
$$\frac{PC - PC_{min} \cdot CA}{PC} \cdot \frac{CC - CC_{min} \cdot CA}{CC} \tag{10}$$

$$\frac{dCA}{dt} = -\frac{\mathrm{F4} \cdot CA}{365 \cdot \mathrm{V}} + (GRW \cdot 0.7 - \mathrm{MA} - \mathrm{SA})CA - \mu Z \cdot \frac{CZ}{\mathrm{F}} \tag{11}$$

$$\frac{dCZ}{dt} = -\frac{\mathrm{F4} \cdot CZ}{365 \cdot \mathrm{V}} + (\mu Z - RZ - \mathrm{MZ})\, CZ - \mu F \cdot \frac{\mathrm{CF}}{\mathrm{F}} \tag{12}$$

$$\frac{dPS}{dt} = \frac{\mathrm{F1} \cdot \mathrm{P1} + \mathrm{F2} \cdot \mathrm{P2} + \mathrm{F3} \cdot \mathrm{P3} - \mathrm{F4} \cdot PS}{365 \cdot \mathrm{V}} - UP + \frac{RZ \cdot PC \cdot CZ}{CA} \tag{13}$$

$$+ \mathrm{K4} \cdot DP \cdot \mathrm{K6}^{(T-20)} + \frac{1.2(PI - PS) - 1.7}{1000} \cdot \frac{T + 273}{280} \cdot \frac{\mathrm{S}}{\mathrm{V}} \tag{13}$$

$$\frac{dDP}{dt} = \mathrm{MA} \cdot PC + \mathrm{MZ} \cdot CZ(PC/CA) - \mathrm{K4} \cdot DP \cdot \mathrm{K6}^{(T-20)} - \frac{\mathrm{F4} \cdot DP}{365 \cdot \mathrm{V}}$$
$$+ \left(\frac{1}{\mathrm{F}} - 1\right) \cdot \mu Z \cdot CZ(PC/CA) - SA \cdot DP + \left(\frac{1}{\mathrm{F}} - 1\right) \cdot \mu F \cdot \mathrm{CF}(PC/CA) \tag{14}$$

$$\frac{dNS}{dt} = \frac{\mathrm{F1} \cdot N1 + \mathrm{F2} \cdot N2 + \mathrm{F3} \cdot N3 - \mathrm{F4} \cdot NS}{365 \cdot \mathrm{V}} - \frac{\mathrm{KD} \cdot NS}{365 \cdot \mathrm{V}} - UN$$
$$+ \frac{RZ \cdot CZ \cdot NC}{CA} + \mathrm{K4} \cdot DN \cdot \mathrm{K6}^{(T-20)} + \frac{4.0 \cdot SN + 0.08}{1000} \cdot \frac{\mathrm{S}}{\mathrm{V}} \cdot \exp(0.151 \cdot T) \tag{15}$$

$$\frac{dDN}{dt} = \mathrm{MA} \cdot NC + \mathrm{MZ} \cdot CZ(NC/CA) - \mathrm{K4} \cdot DN \cdot \mathrm{K6}^{(T-20)} - \frac{\mathrm{F4} \cdot DN}{365 \cdot \mathrm{V}}$$
$$+ \left(\frac{1}{\mathrm{F}} - 1\right)\mu Z \cdot CZ(NC/CA) + \left(\frac{1}{\mathrm{F}} - 1\right)\mu F \cdot \mathrm{CF}(NC/CA) - \mathrm{SA} \cdot DN \tag{16}$$

$$\frac{dPE}{dt} = \frac{\mathrm{SA}(PC + DP)\mathrm{V} \cdot 10^3}{\mathrm{f} \cdot \mathrm{LUL} \cdot \mathrm{S}} - \mathrm{K5} \cdot PE \cdot \mathrm{K6}^{(T-20)} \tag{17}$$

$$\frac{dPI}{dt} = \frac{\mathrm{K5} \cdot PE \cdot \mathrm{K6}^{(T-20)}}{1 - \mathrm{DMU}} - \frac{1.2(PI - PS) - 1.7}{\mathrm{LUL}(1 - \mathrm{DMU})} \cdot \frac{T + 273}{280} \tag{18}$$

$$\frac{dSN}{dt} = \frac{\mathrm{SA}(NC + DN)}{\mathrm{LUL}} \cdot \frac{\mathrm{V}}{\mathrm{S}} - \frac{4.0 \cdot SN + 0.08}{1000 \cdot \mathrm{LUL}} \cdot \exp(0.151 \cdot T) \tag{19}$$

$$\frac{dPC}{dt} = UP - \mathrm{MA} \cdot PC - \mathrm{SA} \cdot PC - \mu Z \cdot \frac{CZ \cdot PC}{\mathrm{F} \cdot CA} - \frac{\mathrm{F4} \cdot PC}{365 \cdot \mathrm{V}} \tag{20}$$

$$\frac{dNC}{dt} = UN - \mathrm{MA} \cdot NC - \mathrm{SA} \cdot NC - \mu Z \cdot \frac{CZ \cdot NC}{\mathrm{F} \cdot CA} - \frac{\mathrm{F4} \cdot NC}{365 \cdot \mathrm{V}} \tag{21}$$

$$\frac{dCC}{dt} = UC - \mathrm{MA} \cdot CC - \mathrm{SA} \cdot CC - \mu Z \cdot \frac{CZ \cdot CC}{\mathrm{F} \cdot CA} - \frac{\mathrm{F4} \cdot CC}{365 \cdot \mathrm{V}} \tag{22}$$

Forcing functions: see Table I

TABLE VI

Parameters

Symbol used	Definition	Value	Found by
CC_{max}	Maximum kg C per kg of phytoplankton biomass	0.6	Estimated values on basis of Stumm and Morgan (1970), Steele (1974), analysis of fractions 1—80 μ, the values for single cells given by Lehmann et al. (1975) related to size of the cells given by Nygaard (1952)
CC_{min}	Minimum kg C per kg of phytoplankton biomass	0.18	
PC_{max}	Maximum kg P per kg of phytoplankton biomass	0.03	
PC_{min}	Minimum kg P per kg of phytoplankton biomass	0.003	
NC_{max}	Maximum kg N per kg of phytoplankton biomass	0.08	
NC_{min}	Minimum kg N per kg of phytoplankton biomass	0.02	
UP_{max}	Increase of kg P per kg phytoplankton biomass per 24 h	0.0035/24 h	Calibration. The ratio UP : UN : UC is in accordance with Lehmann et al. (1975)
UN_{max}	Increase of kg N per kg phytoplankton biomass per 24 h	0.0096/24 h	
UC_{max}	Increase of kg C per kg phytoplankton biomass per 24 h	0.45/24 h	
RA_{max}	Respiration of kg C per kg phytoplankton biomass per 24 h	0.286/24 h	Lehmann et al., (1975) and Nygaard (1952)
GRW_{max}	Cell division rate per 24 h	3/24 h	Lehmann et al., (1975)

All other parameters see Table II

Fig. 1 shows the principles of the model for phosphorus.

The model has the following advantages:

(1) The two steps described — uptake and growth — are in accordance with physiological observations (see also Nyholm, 1975).

TABLE VII

Variables

Symbols	Definition	Unit
PC	P in phytoplankton	mg/l
NC	N in phytoplankton	mg/l
CC	C in phytoplankton	mg/l
C	Inorganic C in water, fixed value 100 mg/l is used	mg/l

All other variables see Table III

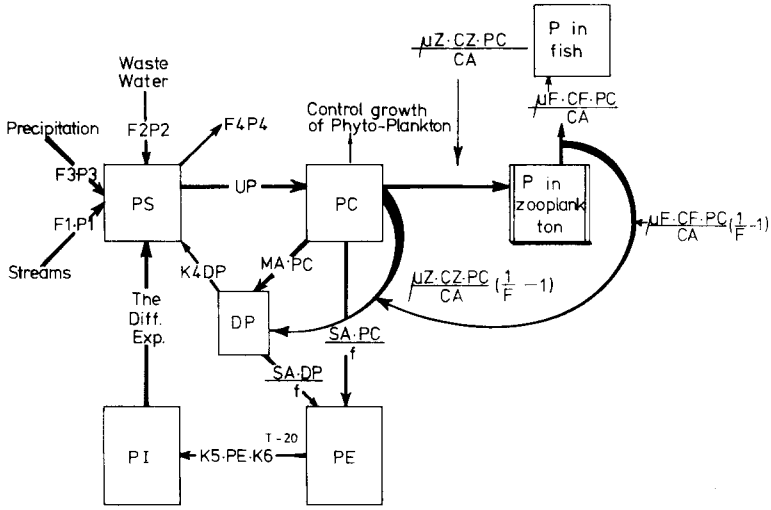

Fig. 1. Principles for the model based upon phytoplankton population dynamics. Only phosphorus is included in this illustration.

(2) The respiration rate is dependent upon the carbon concentration of the cells, which is in accordance with physiological observations.

(3) The parameters PC_{max}, PC_{min}, NC_{max}, NC_{min}, CC_{max} and CC_{min} are in accordance with actually measured compositions of phytoplankton.

RESULTS

The results of model 1 are given in Figs 2—4 where the productivity (g C m^{-2} 24 h^{-1}), phytoplankton (mg biomass/l), and soluble phosphorus (mg/l) are shown.

The values of the model are compared with measured values:

April 1st, 1972—April 1st, 1973, April 1st, 1973—April 1st, 1974 and April 1st, 1974—April 1st, 1975.

The productivity is measured by means of the C^{14} method (see Steemann, 1962, 1965). Since 36% of phytoplankton (dry matter) is carbon (see e.g. Stumm, 1970), and the average depth is 1.8 m, the productivity expressed as g C m^{-2} 24 h^{-1}, is found on the basis of the model as $\mu A \cdot CA \cdot 0.36 \cdot 1.8$.

Phytoplankton (mg biomass/l) is equal to CA. CA has been determined in 1972—1973, by counting the number of cells, multiplied by the average biomass of one cell (Scenedesmus). The size of the cell is taken from Nygaard (1952) and the specific gravity is close to 1.0. In 1973—1974 the phytoplankton biomass was determined by means of the chlorophyll concentration (chlorophyll is $\frac{1}{136}$ of the total biomass; Patten et al. (1975)).

Since October 1974, the phytoplankton biomass has also been determined by weighing the fraction 1-80 μ after filtration of 2 l.

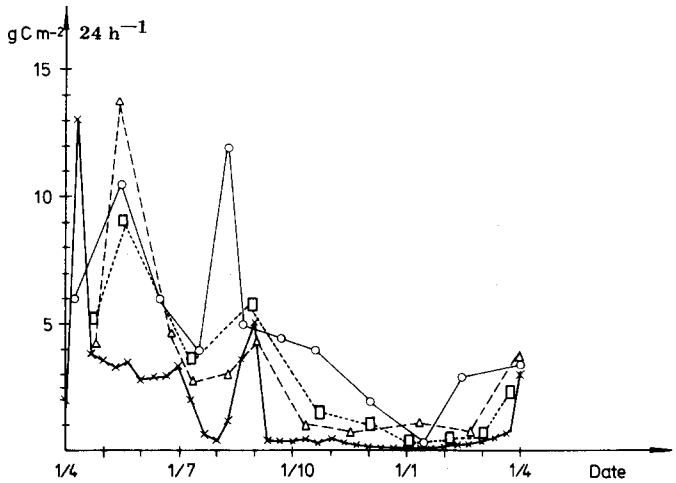

Fig. 2. Production g C/m^2 24 h found by means of model 1 (x) compared with the measured data ○ 1972—1973, △ 1973—1974 and □ 1974—1975.

Figs 5—7 give the result of the second model, again compared with the results from April 1st, 1972—April 1st, 1975: productivity (g C m^{-2} 24 h^{-1}), phytoplankton (mg biomass/l) and soluble phosphorus (mg/l). The productivity is in this case found to be $CC \cdot 0.7\ GRW \cdot 1.8$.

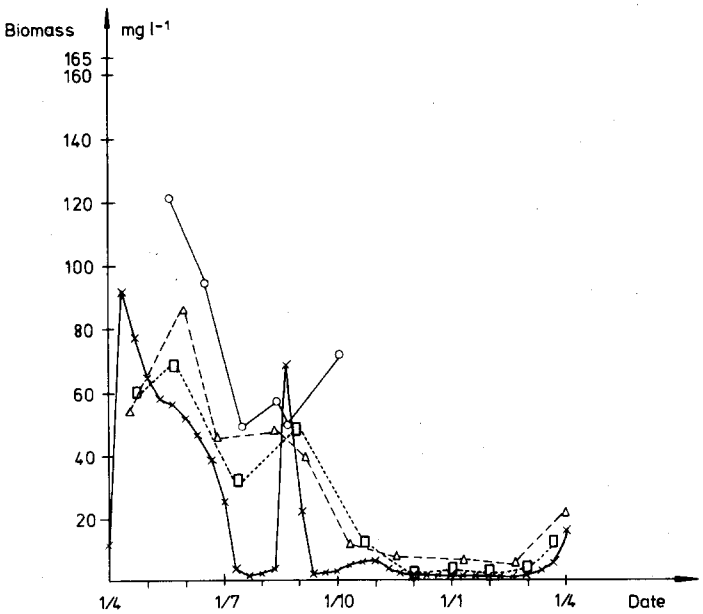

Fig. 3. Biomass (phytoplankton) mg/l found by means of model 1 (x) compared with the measured data ○ 1972—1973 (counting number of cells), △ 1973—1974 (chlorophyll concentration) □ 1974—1975 (chlorophyll concentration).

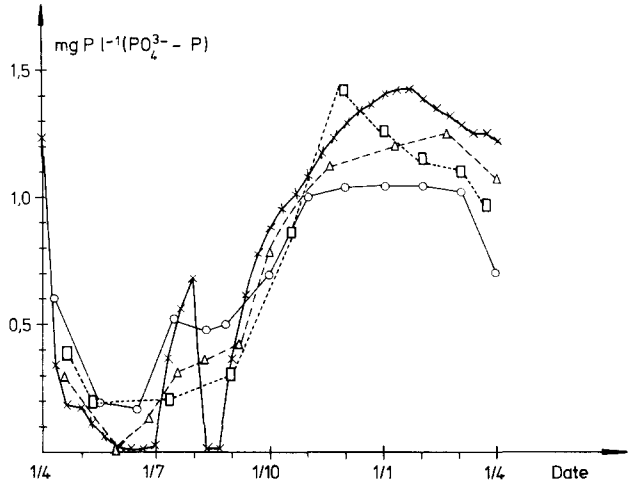

Fig. 4. Soluble phosphorus mg/l found by means of model 1 (x) compared with measured data ○ 1972—1973, △ 1973—1974 and □ 1974—1975.

The zooplankton biomass is determined by counting the number 1972—1973, and since October 1974 by weighing the fraction >80 μ after filtration of 2 l.

All the parametric values determined by measurements are averaged over the period of 3 years, April 1st, 1972—April 1st, 1975. Therefore, the results of the model should rather be compared with the average values measured.

Table VIII shows the maximum production (g C/m^{-2} 24 h) found by means of the model for each month, and the corresponding values measured averaged over the 3 years 1972—1975. The transparency, V, found on the basis of the

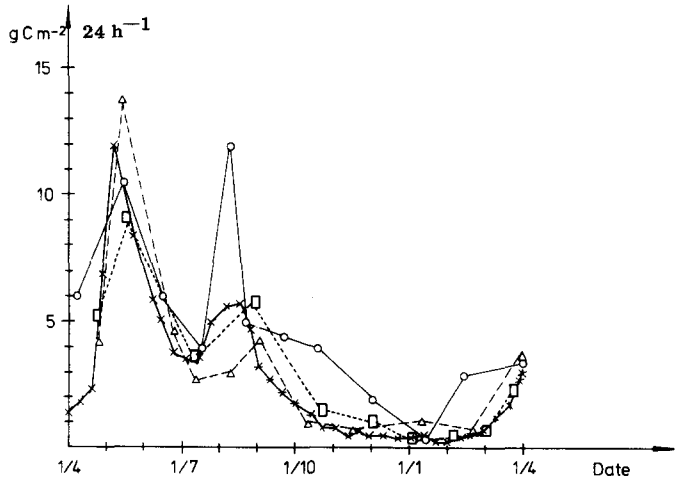

Fig. 5. The production g C/m^2 24 h found by means of model 2 (x) compared with the measured data ○ 1972—1973, △ 1973—1974 and □ 1974—1975.

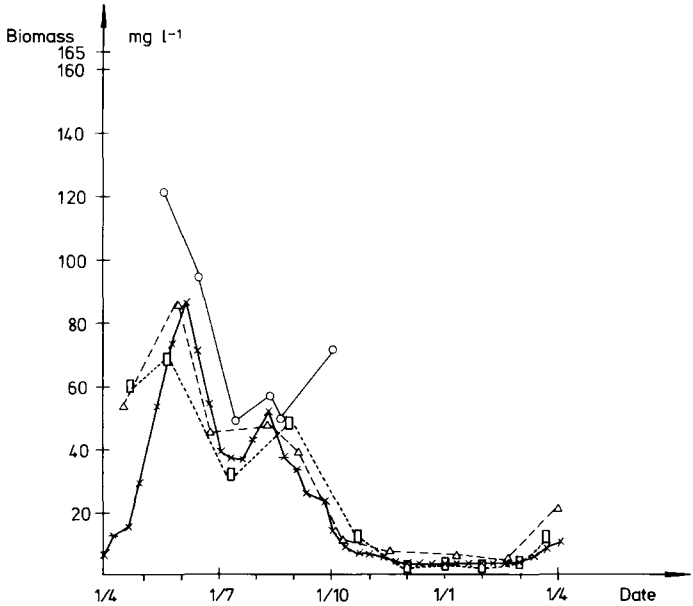

Fig. 6. Biomass (phytoplankton) mg/l found by means of model 2 (x) compared with the measured data ○ 1972—1973 (counting number of cells), △ 1973—1974 (chlorophyll concentration) □ 1974—1975 (chlorophyll concentration).

model, using the relation between V and max g C/m^3 24 h published by Mathiesen (1970) and the transparency measured is shown in Table IX for the summer months (April—September).

The total yearly phytoplankton production found by means of the model

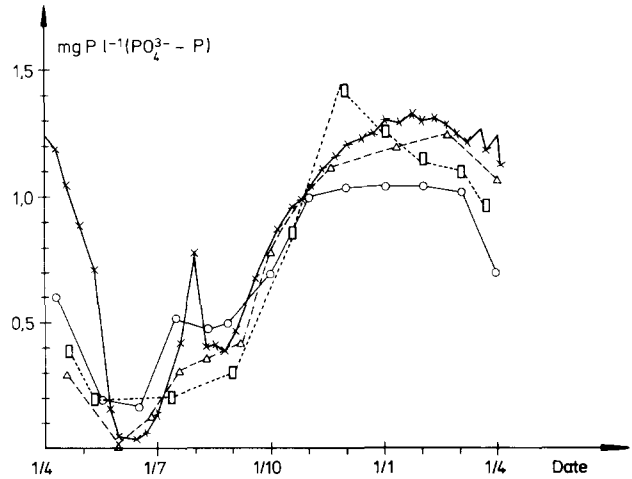

Fig. 7. Soluble phosphorus mg/l found by means of model 2 (x) compared with measured data ○ 1972—1973, △ 1973—1974 and □ 1974—1975.

TABLE VIII

Max g C/m^2/24 h

Month	Model I	Model II	Measured average
April	13	2.4	5.0
May	3.5	12.0	11.2
June	3.0	5.0	5.0
July	3.4	4.8	3.3
August *	5.0	5.7	6.9
September *	5.0	3.2	5.0
October	0.5	1.6	2.2
November	0.5	0.8	1.5
December	0.1	0.4	1.2
January	0.1	0.3	0.4
February	0.3	0.4	1.3
March	2.5	2.8	2.7

* September 1st included.

TABLE IX

Transparency (m)

Month	Model I	Model II	Measured average
April	0.20	0.50	0.35
May	0.45	0.20	0.20
June	0.50	0.30	0.26
July	0.45	0.32	0.31
August	0.30	0.28	0.22
September	0.30	0.48	0.25

and measured values 1972—1973, 1973—1974 and 1974—1975 included the average of the 3 years, and the last 2 years are compared separately in Table X.

DISCUSSION

Both models 1 and 2 are reasonably in accordance with the observed data. However, the productivity determined the first year is considerably higher than the two following years and than the results of the model. The reason for this discordance is probably, that the data from the first years are based upon measurements taken at only two depths, which is insufficient. The following years the determinations were carried out at five different depths: 10 cm, 30 cm, 50 cm, 70 cm and 90 cm (compare with Jørgensen et al., 1973).

The second model seems to be the most in accordance with the measured data. As seen, the maximum productivity and maximum phytoplankton biomass both fall at approximately the right time. Furthermore, model 2 gives a yearly C-production closer to the measured values, see Table X. As model 2

TABLE X
Comparison of measured and calculated production values

Total Production	g C/m^2/year
Model I	850
Model II	1,050
Measured 1972—1973	1,800
Measured 1973—1974	1,150
Measured 1974—1975	1,050
Measured average 1972—1973	1,330
Measured average 1973—1975	1,100

also has other advantages (see above), it will be preferred for later uses of the lake model.

As mentioned above the measured parametric values and forcing functions are based upon average values for the period April 1st, 1972—April 1st, 1975. The outputs of the models could also have been based upon values from each individual year, but since the models will be used for environmental impact statements, their goal was to give a picture of the average year rather than of a specific year. Furthermore, a comparison of the deviations between the models and the observations with the deviations between observations from year to year is now possible. If the productivity determinations for the first year are omitted, the deviation between model 2 and the observations between year to year observations, while model 1 gives approximately the same deviation from the observations as model 2, but the maxima come too early in the year. Figs 8 and 9 show the results of a waste-water treatment, which besides the existing mechanical-biological treatment, will remove most of the phosphorus from the waste water. Two cases have been studied:

(A) The treated waste water discharged has a concentration of 0.4 mg/l.

(B) The treated waste water discharged has a concentration of 0.1 mg/l.

The model has also been tested in the case nitrogen is removed from the waste water, but as approximately one-third of the nitrogen input to the lake comes from natural sources, the improvement of the water quality was modest.

Table XI gives the expected productivity and transparency 3 and 10 years after the initiation of the phosphorous removal.

Already after 3 years both models show a significant improvement of the water quality mainly as a result of the use of the submodel for exchange of nutrient between sediment and water.

Conditions prevailing after 3 and 10 years, if a first order reaction for the exchange process were used without distinguishing between exchangeable and unexchangeable phosphorus, has been estimated. These estimations are included in Table XI, and as seen, it gives a considerably slower improvement of the lake.

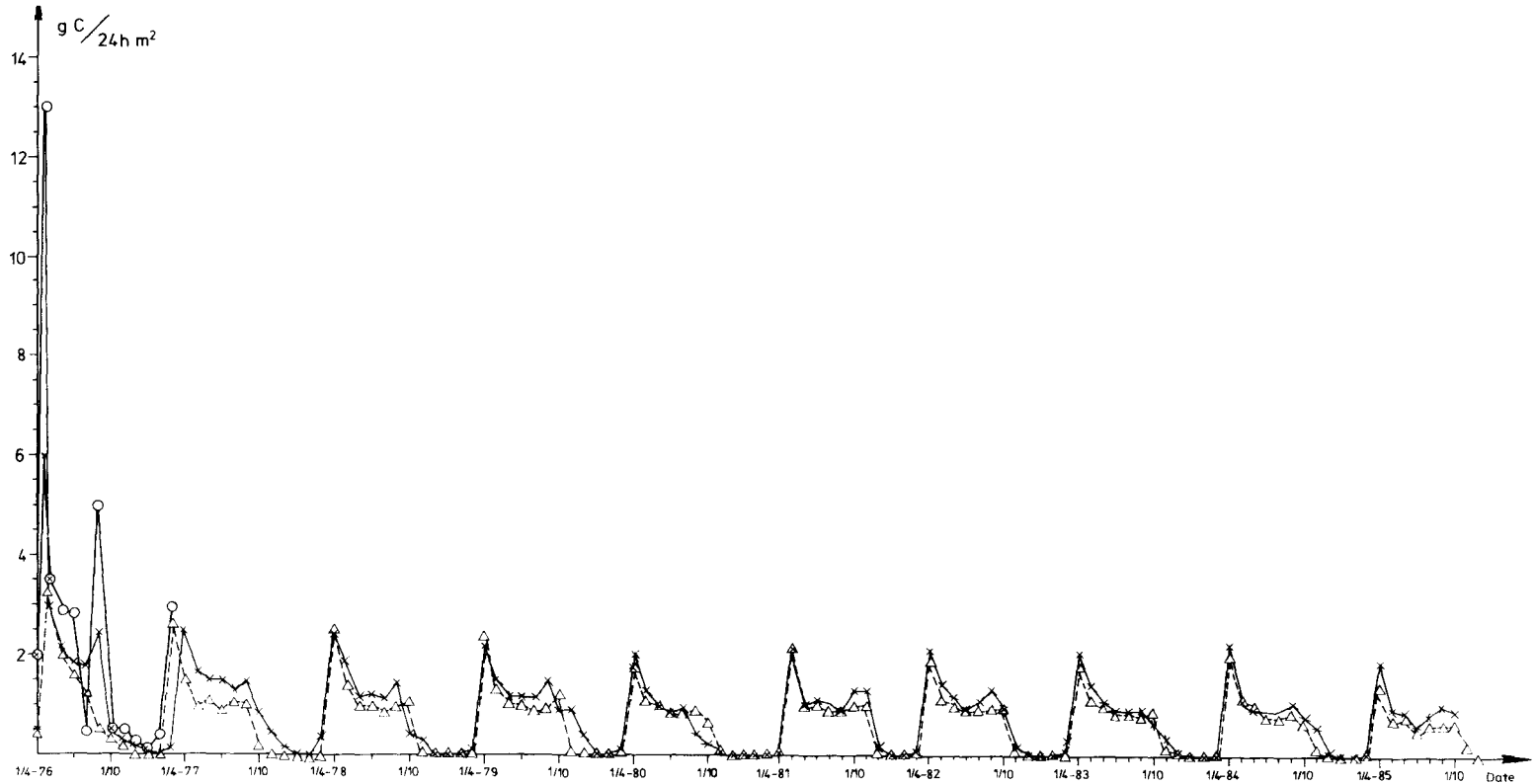

Fig. 8. Productivity g C/m^2 24 h in case (a) phosphorus is not removed from the waste water ○; (b) waste water with 0.4 mg P/l is discharged (x); (c) waste water 0.1 mg P/l is discharged △. All data based upon model 1.

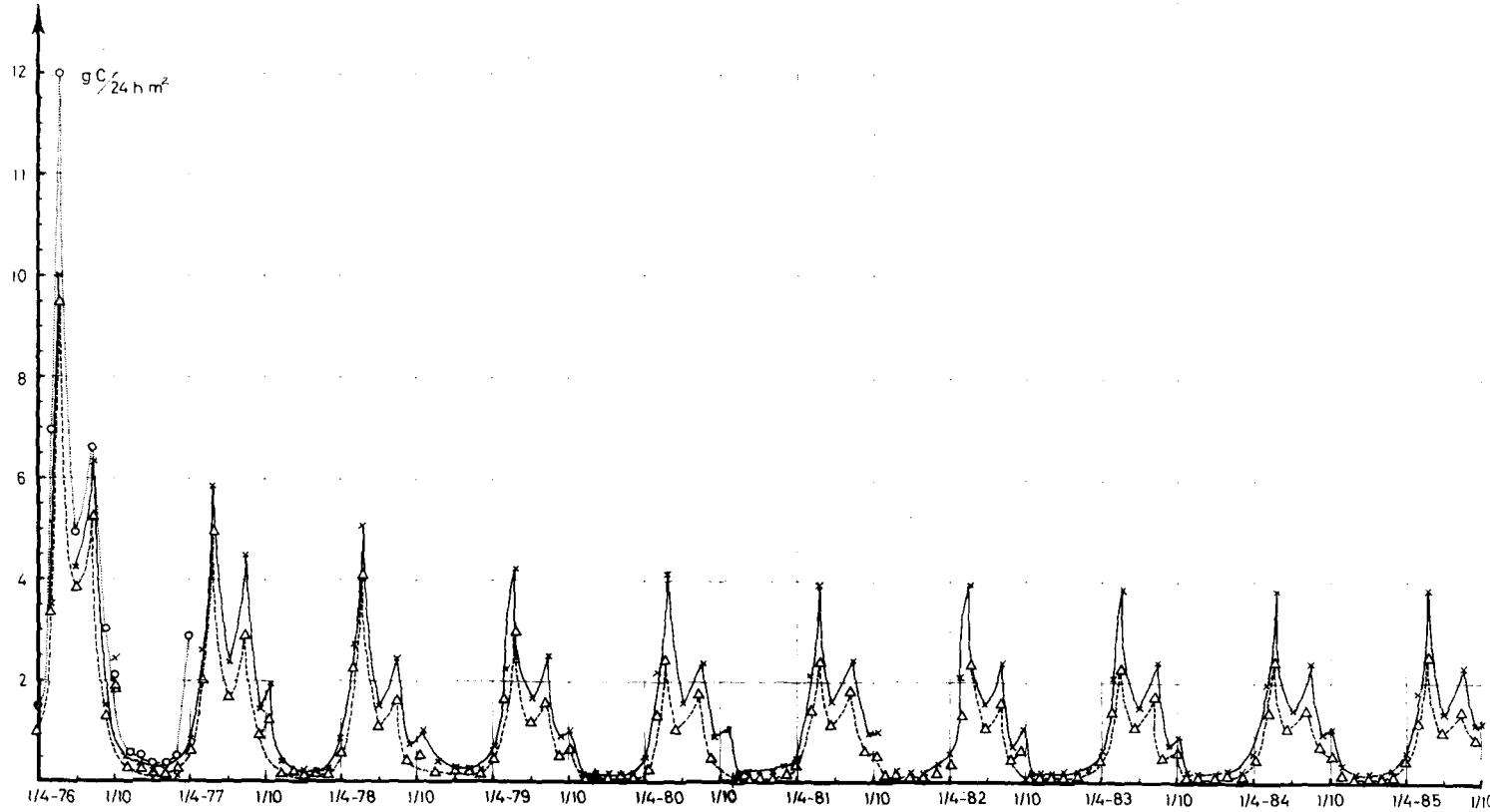

Fig. 9. Productivity g C/m^2 24 h in case (a) phosphorus is not removed from the waste water ○, (b) waste water with 0.4 mg P/l is discharged (x), (c) waste water 0.1 mg P/l is discharged △. All data based upon model 2.

TABLE XI

Predictions by means of the models in two cases: concentration of treated waste water: A: 0.4 mg P/l; B: 0.1 mg P/l

Model I:	3rd year		After 9th year	
	Case A	Case B	Case A	Case B
g C/m^2/year	340	300	280	240
Minimum transparency (cm)	70	80	90	100
Model II:				
g C/m^2/year	650	500	500	320
Minimum transparency (cm)	50	60	60	75
First order reaction P-sediment water				
g C/m^2/year	850	800	610	480
Minimum transparency (cm)	30	35	50	60

CONCLUSIONS

Application of ecological models for environmental impact statements is in its infancy. The results of this work must therefore be regarded as tentative. The model is a simplification of the ecosystem, and our knowledge of the significance of these simplifications is not yet sufficient.

However, on the basis of the results, model 2 must be preferred to model 1, as it is more in accordance with the observations, and as it is in accordance with knowledge of the physiology of the phytoplankton. This last feature must be strongly emphasised.

At least for the lake examined, it seemed necessary to include in the models the seven points mentioned below:

(1) The submodel for the nutrient exchange rate from sediment to water. Separate calibration of model has been a great advantage.

(2) Three trophic levels.

(3) Denitrification.

(4) An exponential expression for the influence of the temperatures on the growth of plankton and fish.

(5) To use a forcing function for the nitrogen input.

(6) The light equation (20) Table I.

(7) A threshold concentration of phytoplankton for the zooplankton grazing rate.

However, examinations over the coming years, after the phosphorous removal is in operation, are needed to demonstrate the significance of these points.

Whether a more complicated model including more species, benthic animals, macrophytes etc. should be used is also to be asked.

Furthermore, some of the essential variables can be determined only indirectly, e.g. phytoplankton biomass is determined by means of chlorophyll, and the ratio chlorophyll to the biomass varies. Therefore, the determination of phytoplankton biomass by means of chlorophyll must be taken as a rough estimate.

The two models set up are probably the first two generations in a series of models. Some of the equations have not been sufficiently examined, such as the equations on grazing, decomposition of the organic nitrogen and phosphorus, while other processes are based upon a knowledge of main processes and their parameters.

ACKNOWLEDGEMENT

I wish to thank Mogens Friis, Eng. for his assistance in working out the EDB-program.

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