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# Optimal control as a tool to investigate the profitability of a Chinese plant factory - lettuce production system



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Keywords: Optimal control Plant factory LED light intensity Dark period Profitability Although optimal control has been used extensively for greenhouse climate management, its application to plant factories is still in its infancy. In this paper, profitability of growing lettuce in a Chinese plant factory is investigated by means of optimal control computations. To that end, first, a lettuce growth model is adapted to fit a plant factory environment. Next, this model is calibrated and validated using nine sets of experimental data with different LED light intensities. Using the calibrated and validated model, optimal control computations are used to produce a 3D plot revealing the influence of the electricity and lettuce price on maximum profit. Lettuce's physiological demand for dark periods during artificial lighting is incorporated by fixing this dark period to eight hours a day. Therefore, the optimal LED light intensity pattern obtained from the optimal control computations could be used in the actual production process. Maximum profit can reach 264.88 RMB m<sup>-2</sup> assuming a Chinese plant factory fresh lettuce price of 34.5 RMB kg<sup>-1</sup>. When lettuce must be sold at 5.01 RMB  $kg^{-1}$ , which represents the price for an open field product, profit always comes out negative. Besides, LED lighting is not advised when the electricity price is greater than 0.84 RMB kWh<sup>-1</sup> under these circumstances. Profit is only positive when the lettuce price is over 20 RMB  $kg^{-1}$ .

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Nomencla	Nomenclature							
Symbol	Physical meaning	Value	Unit					
X <sub>d</sub>	Lettuce dry mass		kg[dw] m <sup>-2</sup>					
t	Time		S					
$C_{\alpha\beta}$	Yield factor	0.544	kg[CO <sub>2</sub> ] kg[dw] <sup>-1</sup>					
$\varphi_{phot,c}$	Gross canopy photosynthesis rate		$kg[CO_2] m^{-2} s^{-1}$					
C <sub>resp,d</sub>	Respiration rate	$2.65 \cdot 10^{-7}$	s <sup>-1</sup>					
X <sub>T</sub>	Air temperature in the greenhouse		°C					
C <sub>pl,d</sub>	Effective canopy surface	53	$m^2 kg[dw]^{-1}$					
C <sub>rad,phot</sub>	Solar light use efficiency	$3.55 \cdot 10^{-9}$	kg[dw] J[PAR] <sup>-1</sup>					
V <sub>rad</sub>	Solar radiation outside the greenhouse		$W[PAR] m^{-2}$					
C <sub>co2,1</sub>	Temperature effect on CO <sub>2</sub> diffusion in leaves	$5.11 \cdot 10^{-6}$	m s <sup>-1</sup> $^{\circ}C^{-2}$					
C <sub>co2,2</sub>	Temperature effect on CO <sub>2</sub> diffusion in leaves	$2.30 \cdot 10^{-4}$	$\mathrm{m}~\mathrm{s}^{-1}~\mathrm{^{\circ}C^{-1}}$					
C <sub>co2,3</sub>	Temperature effect on CO <sub>2</sub> diffusion in leaves	$6.29 \cdot 10^{-4}$	${ m m~s^{-1}}$					
Xc	Carbon dioxide concentration in greenhouse		kg[CO <sub>2</sub> ] m <sup>-3</sup>					
CΓ	Carbon dioxide compensation point	$5.2 \cdot 10^{-5}$	$kg[CO_2] m^{-3}$					
ε	Solar light use efficiency	17·10 <sup>-9</sup>	kg[dw] J[PAR] <sup>-1</sup>					
Cpar	The ratio of photosynthetically active radiation to total solar radiation	0.5						
C <sub>rad,rf</sub>	The transmission coefficient of the roof for solar radiation	0.42						
P <sub>1</sub>	Profit in the first optimal control formulation		$RMB m^{-2}$					
P <sub>2</sub>	Profit in the second optimal control formulation		$RMB m^{-2}$					
Clettuce	Price per lettuce fresh mass	5.01-34.5	RMB kg[fw] <sup>-1</sup>					
Cenergy	Price of electric energy in agriculture	0.30-0.92	RMB kWh[E] <sup>-1</sup>					
t <sub>o</sub>	Start time	0	S					
t <sub>f</sub>	End time	1.8144 · 10 <sup>6</sup>	S					
C <sub>cost</sub>	Costs without LED lighting	$4.20 \cdot 10^{-5}$	$\rm RMB~m^{-2}~s^{-1}$					
Ck	The extinction coefficient of the canopy	0.9						
C <sub>lar,d</sub>	The shoot leaf area ratio (calibrated)	24.63	$m^2 kg[dw]^{-1}$					
$C_{\tau}$	The ratio of the root dry mass to the lettuce dry mass (calibrated)	0.084						
C <sub>pl,d</sub>	Effective canopy surface (calibrated)	29.29	$m^2 kg[dw]^{-1}$					
C <sub>αβ</sub>	Yield factor (calibrated)	0.51	kg[CO <sub>2</sub> ] kg[dw] <sup>-1</sup>					
Cled,phot	LED light use efficiency (calibrated)	12.06 · 10 <sup>-9</sup>	kg[dw] J[PAR] <sup>-1</sup>					
U <sub>led</sub>	Controllable LED light intensity		$W[PAR] m^{-2}$					
C <sub>cap,c</sub>	The volumetric capacity of plant factory air for CO <sub>2</sub>	2.5	m[CO <sub>2</sub> ] <sup>3</sup> m <sup>-2</sup>					
Cresp,c	Respiration rate in terms of produced CO <sub>2</sub>	$4.87 \cdot 10^{-7}$	$s^{-1}$					
Uc	The supply rate of CO <sub>2</sub>		$kg[CO_2] m^{-2} s^{-1}$					
C <sub>co2</sub>	Cost of CO <sub>2</sub>	4	RMB kg[CO <sub>2</sub> ] <sup>-1</sup>					
C <sub>fw</sub>	Fresh mass / dry mass ratio (calibrated)	20.98						
C <sub>trans</sub>	The transfer coefficient from electricity to LED light intensity	60.6%						
C <sub>unit</sub>	The transfer coefficient from unit kWh to J	3.6·10 <sup>6</sup>						
U <sub>ledmax</sub>	The upper bound of LED light intensity	140 during 06:00-22:00 hours 0	$W[PAR] m^{-2}$					
		during 22:00-06:00 hours						
X <sub>cmax</sub>	The upper bound of CO <sub>2</sub> concentration	1400	ppm					
U <sub>cmax</sub>	The upper bound of CO <sub>2</sub> supply rate	$1.2 \cdot 10^{-6}$	$kg[CO_2] m^{-2} s^{-1}$					
n <sub>p</sub>	Number of simultaneously estimated parameters in an identifiability	2	·					
	analysis							
[dw], [fw] [	CO <sub>2</sub> ] [PAR] and [E] represent dry mass fresh mass CO <sub>2</sub> photosynthetically	v active radiation, and energy respec	rtively					

# 1. Introduction

With the limited availability of water and mineral nutrients as well as reduced availability of labour and accessibility to land and fertile soil, the increasing urban population demands higher food production and production efficiency (Pennisi et al., 2019). A plant factory, or 'vertical farm', provides a promising prospect. It increases the efficiency of water and nutrient use through control of environmental factors and by limiting exchanges with the external environment (Benke & Tomkins, 2017; Shamshiri et al., 2018; Beacham et al., 2019; O'Sullivan et al., 2019). A simulation study with lettuce showed that vertical farms can increase the efficiency of land, water and nutrient use when compared to greenhouses located in Sweden, the Netherlands, or the United Arab Emirates (Graamans et al., 2018). However, the high cost of greater energy requirement, mainly associated with electric lighting, is not negligible. According to Yang (2019), the energy cost of artificial lighting accounts for 60% of the total energy cost in a plant factory. Therefore, optimal use of artificial lighting remains a crucial factor in promoting plant factory production.

Although optimal control has been widely researched in greenhouses (González et al., 2014; Kang et al., 2018; Lin et al., 2020; Seginer et al., 2017; Van Beveren et al., 2015; Van Straten et al., 2011; Xu et al., 2018a), its application to plant factories is still in its infancy (Avgoustaki & Xydis, 2020). Some researchers considered possible benefits of supplemental lighting in greenhouses through some form of optimal control (Ioslovich, 2009; Xu et al., 2018b, 2019). However, studies on optimal control often result in continuous supplemental lighting. According to Hernandez et al. (2020), supplemental lighting might increase tip-burn occurrence in greenhouse lettuce. Tip-burn also occurs in plant factory lettuce production (Ahmed et al., 2020b). Moreover, research also shows that lettuce does not grow proportionally with the increase of continuous artificial lighting in a plant factory (Pennisi et al., 2020; Zha et al., 2019). Therefore, certain dark periods are necessary for plant factory lettuce production (Ahmed et al., 2020a). Unfortunately, most lettuce growth models lack mechanisms that reflect the need for dark periods for proper growth (Ioslovich, 2009; Van Henten, 1994). Since, without artificial lighting, dark periods occur naturally, optimal control computations without artificial lighting do not suffer from this lack. Optimal control of plant factories, on the other hand, incorporates artificial lighting and may suffer from it. This explains the occurrence of optimal continuous artificial lighting (Xu et al., 2018b, 2019). To improve on this, in this paper dark periods are enforced in our optimal control computations by switching the upper bound of LED lighting to zero for eight hours each night. This dark period of eight hours is selected because a photoperiod of sixteen hours per day is thought to be the most energy efficient (Pennisi et al., 2020).

According to Bergstrand et al. (2016), research conclusions on lettuce responses to artificial light can be contradictory. This phenomenon results from differences in lettuce variety, experimental settings, nutrient solution, and so on (Loconsole et al., 2019). Therefore, in this paper we develop, calibrate and validate a lettuce crop model for use under plant factory conditions. The calibration and validation rely on nine sets of experiments with three different light intensities but a fixed red/blue light intensity ratio of 4:1. Initial work on this topic was based on one set of experiments with a red/blue light intensity ratio of 9:1 (Xu et al., 2020). One should note that lettuce shows different responses under different light spectra (Wang et al., 2016). Only one parameter was calibrated in the previous paper while six parameters are calibrated and validated in this paper. These are yield factor  $c_{\alpha\beta}$ , LED light use efficiency  $c_{led,phot}$ , effective canopy surface c<sub>pl,d</sub>, shoot leaf area ratio c<sub>lar,d</sub>, ratio of root dry mass to lettuce dry mass  $c_{\tau}$ , and fresh mass/dry mass ratio  $c_{fw}$ . Next, as in our initial work on this topic, the calibrated and validated model is used to perform optimal control computations to investigate the profitability when lettuce is grown in a plant factory and artificial LED light intensity is optimised. Specifically, maximum profit is computed given different prices for electricity used by LED artificial lighting and two prices for selling lettuce. A high price for lettuce can be obtained if crop quality is high, as in plant factories. A much lower price is obtained for lettuce grown in the open field, which have much-reduced quality.

This paper further extends our initial work in four different ways. Firstly, this paper presents a 3D plot of maximum profit against continuous variations of both the electricity and lettuce price. Secondly, comparisons between optimal control and conventional setpoint control are presented. Thirdly, cooptimisation of LED lighting and CO<sub>2</sub> supply is presented showing that the optimal control approach allows for extension with environmental factors and control inputs when models describing their influence are available. Finally, part of the lettuce model calibration procedure is the simultaneous estimation of two of the six parameters using nonlinear least squares. An important prerequisite is that both parameters are identifiable. This is investigated by means of a highly efficient algorithm (Stigter & Molenaar, 2015; Xu et al., 2018a).

As in our previous work, by enforcing a dark period of eight hours each night, the optimal LED light intensity patterns consider unmodelled physiological reactions of lettuce requiring dark periods. Therefore, the optimal LED light intensity patterns can be used in the actual production process as opposed to using continuous LED lighting (Xu et al., 2018b, 2019).

### 2. Materials and methods

#### 2.1. Experimental setup

Nine experiments intended for calibration and validation of the lettuce growth model under plant factory conditions were conducted at the Chinese Academy of Agricultural Sciences (CAAS), Beijing, China in one compartment of a 4 m  $\times$  2.2 m  $\times$  2.5 m plant factory with artificial light. The plant factory contained four three-layered culture shelves (1.5 m  $\times$  0.7 m  $\times$  2.4 m) with each two lined against the wall. The culture beds were equipped with vertically movable light-emitting diodes (LEDs) panels (Dongguan Bio-lighting Sciences and Technology Co. Ltd, China) with a maximum output at a wavelength of 660 nm and 450 nm for the red and blue light, respectively. The vertical distance between the LEDs and the culture beds was fixed at 0.3 m. The photon flux density of the red and blue lights was controlled by adjusting the DC power supply (PKU-MS605D). By continuously adapting the DC voltage, optimal controls can be implemented. Plants on each culture bed were placed on a portable Styrofoam that floated above the nutrient solution that is circulated in a deep flow technique (DFT) way. A heat pump with a cooling capacity of 14 kW (HFW-75-2, Beijing, China) was used to control the air temperature and relative humidity. CO2 was supplied using a CO<sub>2</sub> gas cylinder (control of gas flow) and sensed by an infrared CO<sub>2</sub> probe (ZFP, Fuji Electric Co. Ltd., Tokyo, Japan).

The lettuce variety 'Tiberius' was chosen as the research object, and 32 plants  $m^{-2}$  were cultivated on the culture beds with 1-hour nutrient solution circulation per day. The nutrient solution was taken from the CAAS experimental base in Shunyi for leafy vegetable cultivation. From 19th July to 11th November in 2019, 9 experiments with different constant LED light intensities and CO<sub>2</sub> concentrations were performed, as shown in Table 1. Each cultivation took 21 days with a red/blue LED light intensity ratio of 4:1 and a photoperiod of 16 h day<sup>-1</sup>. The photoperiod of 16 h day<sup>-1</sup> is thought to be the most energy efficient (Pennisi et al., 2020). Note that the effect of changes in

Table 1 — Environmental setpoints during light periods and experiment/dataset numbers.							
LED light intensity (µmol $m^{-2} s^{-1}$ ) CO <sub>2</sub> concentration ppm	100	200	300				
500	1	2	3				
1000	4	5	6				
1500	7	8	9				

the light spectra on lettuce growth is considered out of the scope of this research. Note that temperature, relative humidity, CO<sub>2</sub> concentration, and air velocity were controlled by conventional setpoint controllers following fixed daily patterns according to a commonly used plant factory setup (Ahmed et al., 2020b; Zha et al., 2019). The numbering of the 9 experiments and the corresponding combinations of CO2 concentration and LED light intensity during light periods are shown in Table 1. The other environmental setpoints were constant at 24 °C for air temperature, 70% for relative humidity, and 0.5 m s<sup>-1</sup> for air velocity. To illustrate the proper performance of the conventional setpoint controllers, the average measured temperature from two sensors located at different points is shown in Fig. 1. Although slightly oscillating, the average temperature stays around the setpoint of 24 °C. The slightly higher temperature during the light period is caused by the small amount of heat released by the LED lighting devices during the photoperiod of 16 h and the fact that the two sensors for temperature measurement are different from the temperature sensor used by the controller.

Within each of the 9 experiments, at the start of every week, 4 plants were randomly sampled for 1) leaf fresh mass, 2) leaf dry mass, 3) root fresh mass, 4) root dry mass, and 5) leaf area measurement, except for the last harvest at day 21. During this last harvest, 8 plants were randomly sampled. Table 2 represents these measurements performed in a single experiment. Each experiment thus contains  $4 \times 5 + 4 \times 5 + 8 \times 5$  being 100 measurements. For all 9 experiments, the total number of measurements thus becomes 900.

The dry mass measurements were performed after a destructive oven-drying treatment. The leaf area measurements were performed using the leaf area meter LI-3100 from LI-COR Inc., Nebraska, USA.

#### 2.2. Dynamic lettuce growth model

Our dynamic model of lettuce growth is a slightly modified version of the one for a greenhouse given by Van Henten (2003):

$$\frac{dX_d}{dt} = c_{\alpha\beta}\varphi_{phot,c} - c_{resp,d}X_d 2^{(0.1X_T - 2.5)}$$
(1)

$$\varphi_{phot,c} = \left(1 - e^{-c_{pl,d}X_d}\right) \frac{c_{rad,phot}V_{rad} \left(-c_{co_{2,1}}X_T^2 + c_{co_{2,2}}X_T - c_{co_{2,3}}\right) (X_c - c_{\Gamma})}{c_{rad,phot}V_{rad} + \left(-c_{co_{2,1}}X_T^2 + c_{co_{2,2}}X_T - c_{co_{2,3}}\right) (X_c - c_{\Gamma})}$$
(2)



Fig. 1 - Temperature during one day.

Table 2 – Description and number of measurements	
performed within a single experiment.	

Measured quantity $\smallsetminus$ At day	1	7	14	21
Leaf fresh mass	4	4	4	8
Leaf dry mass	4	4	4	8
Root fresh mass	4	4	4	8
Root dry mass	4	4	4	8
Leaf area	4	4	4	8

Physical meanings, values, and units appearing in this model are shown in the nomenclature. Note that  $X_d$  is the single state variable representing lettuce dry massm<sup>-2</sup>. The effect of air humidity is not explicitly considered in this model. This is acceptable if humidity is in a range suitable for crop growth (Van Henten, 2003). The latter is guaranteed by selecting a setpoint of 70% for relative humidity in the plant factory.

In Equation (1), the increase of lettuce dry mass over time  $\frac{dX_d}{dt}$  is given by the increase due to photosynthesis  $c_{\alpha\beta}\varphi_{phot,c}$  and the decrease due to respiration  $c_{resp,d}X_d2^{(0.1X_T-2.5)}$ . Here,  $c_{\alpha\beta}$  is a yield factor,  $\varphi_{phot,c}$  is the gross canopy photosynthesis rate,  $c_{resp,d}$  is the respiration rate expressed in terms of the amount of respired dry mass and  $X_T$  is the air temperature in the greenhouse.

In Equation (2),  $c_{pl,d}$  is the effective canopy surface,  $c_{rad,phot}$  is the light use efficiency,  $V_{rad}$  is the solar radiation outside the greenhouse and  $c_{co_{2,1}}$ ,  $c_{co_{2,2}}$ ,  $c_{co_{2,3}}$  parameterise the temperature influence on gross canopy photosynthesis. Furthermore  $X_c$  is the carbon dioxide concentration in the greenhouse and  $c_T$  is the carbon dioxide compensation point.

According to Van Henten (1994), the light use efficiency  $c_{rad,phot}$  can be represented by Equation (3) where e is the solar light use efficiency and  $c_{par}$  is the ratio of photosynthetically active radiation to total solar radiation. Finally,  $c_{rad,rf}$  is the transmission coefficient of the roof for solar radiation so that,

$$c_{rad,phot} = \varepsilon c_{par} c_{rad,rf} \tag{3}$$

In a plant factory,  $c_{rad,rf}$  becomes 1 because the LED light falls directly on the plant. The value of  $c_{par}$  also becomes 1 because LED light is a mixture of red and blue, both being photosynthetically active radiation.  $V_{rad}$  in Equation (2) changes into  $U_{led}$ , which represents the controllable LED light intensity. Therefore,  $U_{led}$  has the same unit as  $V_{rad}$ . The brightness of the LED can be controlled through a real-time optimal controller by changing the power supply to the LED. Without an optimal controller, one can set the brightness manually each day, before the light period starts. A good practice would be to set this value to the average of the optimal LED lighting pattern during that day. To represent the LED light use efficiency,  $c_{rad,phot}$  was changed into  $c_{led,phot}$ . After modification, the final form of gross canopy photosynthesis rate  $\varphi_{phot,c}$  is given by,

$$\rho_{\text{phot},c} = \left(1 - e^{-c_{\text{pl},d}X_d}\right) \frac{c_{\text{led},\text{phot}}U_{\text{led}}\left(-c_{\text{co}_{2,1}}X_T^2 + c_{\text{co}_{2,2}}X_T - c_{\text{co}_{2,3}}\right)(X_c - c_r)}{c_{\text{led},\text{phot}}U_{\text{led}} + \left(-c_{\text{co}_{2,1}}X_T^2 + c_{\text{co}_{2,2}}X_T - c_{\text{co}_{2,3}}\right)(X_c - c_r)}$$
(4)

To realise the physiological demand for dark periods required by lettuce, the upper bound  $U_{ledmax}$  of the control input  $U_{led}$  is set to zero during a fixed period of eight hours each night from 22:00 until 06:00 (Pennisi et al., 2020; Xu et al., 2020).

#### 2.3. Dynamic CO<sub>2</sub> model

The concept behind a plant factory is that it should be fully closed with all environmental factors controlled artificially. In actual practice,  $CO_2$  gas leakage cannot be completely avoided. Modelling this leakage through experimental data is difficult and out of the scope of this paper. Assuming the plant factory to be fully closed, the dynamics of the state  $X_c$  representing  $CO_2$  concentration is adjusted from Van Henten (2003),

$$\frac{dX_{c}}{dt} = \frac{1}{c_{cap,c}} \left[ -\varphi_{phot,c} + c_{resp,c} X_{d} 2^{(0.1X_{T}-2.5)} + U_{c} \right]$$
(5)

In Equation (5),  $c_{cap,c}$  is the volumetric carbon dioxide capacity of the greenhouse air,  $c_{resp,c}$  is the respiration coefficient expressed in terms of the amount of carbon dioxide produced while  $U_c$  is the supply rate of carbon dioxide.

#### 2.4. Optimal control problem formulation

Taking the growers' economic perspective to maximise profit, two optimal control problems are formulated in this section. One problem considers the crop growth dynamics Equation (1), Equation (4) and has LED light intensity  $U_{led}$  as the single control variable. The other considers in addition the CO<sub>2</sub> dynamics Equation (5) having the CO<sub>2</sub> supply rate  $U_C$  as a second control variable. Plant factory air temperature  $X_T$  is set to be constant at 24 °C in the first optimal control problem as shown in Table 1. Plant factory CO<sub>2</sub> concentration  $X_c$  is set to be constant at 1000 ppm in the first optimal control problem as shown in Table 1. It is regarded as a state variable in the second optimal control problem.

A fixed growing period of 21 days was selected because this is the common plant factory experimental setup for proper lettuce growth (Ahmed et al., 2020b). Also, scheduling arrangements concerning the delivery of lettuce to sellers or customers generally demand a fixed harvest time. Alternatively, the market may demand harvested lettuce to have a fixed mass as assumed by Ioslovich (2009). This turns the optimal control problem into one with a free final time that can also be solved. However, vegetables are mostly sold by weight in China while their sizes may vary a lot.

Complying with optimal control notation and terminology, system dynamics are represented by a set of first order differential equations,

$$\frac{dx}{dt} = f(x, u, t) \tag{6}$$

in which column vector x represents the state containing all state variables and column vector *u* represents the control containing all control variables. Furthermore,

$$t_0, t_f$$
 (7)

represent the initial and terminal time determining the control horizon and

$$J(u(t), x(t_0))$$
 (8)

the cost functional which depends on the initial state

$$x(t_0)$$
 (9)

and the control

$$u(t), t_0 \le t \le t_f \tag{10}$$

In addition, the control variables in Equation (10) may be constrained, as they are by upper and lower bounds in our two optimal control problems, formulated next. Finally, our second optimal control problem formulation will also have a single state variable that is upper bounded, representing a state constraint.

Using the above notation and terminology, a general optimal control problem formulation reads as follows. Given the system dynamics Equation (6), the control horizon Equation (7), and the initial state Equation (9), find the control Equation (10) that maximises the cost functional Equation (8), while satisfying possible control and state constraints.

Given this general formulation, our first optimal control problem formulation is given by,

$$\begin{array}{l} f(x,u) = \ c_{\alpha\beta} \phi_{phot,c} - c_{resp,d} X_d 2^{(0.1X_T-2.5)}, \ x = X_d, \ u = U_{led} \\ t_0 = 0, \ t_f = 21 \times 24 \times 60 \times 60, \ x(t_0) = 1.36 \cdot 10^{-3} \\ 0 \leq u(t) \leq U_{ledmax} \end{array} \tag{11}$$

$$J_{1} = c_{lettuce}c_{fw}x(t_{f}) - \int_{t_{0}}^{t_{f}} \left(\frac{c_{energy}}{c_{unit}c_{trans}}u(t) + c_{cost}\right)dt$$
(12)

In Equation (12),  $J_1$  represents profit to be maximised by the control u(t),  $t_0 \le t \le t_f$ , where  $c_{lettuce}$  is the price per lettuce fresh mass,  $c_{fw}$  is the fresh mass/dry mass ratio and calibrated to 20.98,  $c_{energy}$  is the price of electric energy in agriculture and  $c_{unit}$  is the transfer coefficient from kWh to Joule. Finally,  $c_{cost}$  is a constant that, except for energy required for LED lighting, represents all other costs of growing lettuce in a plant factory such as costs associated with air-conditioning, cultivation, human labour, maintenance, rent, and degradation. According to a cost analysis from a plant factory company in Beijing

$$\begin{split} f(x,u) &= \begin{bmatrix} c_{\alpha\beta} \phi_{phot,c} - c_{resp,d} x_1 2^{(0.1X_T-2.5)} \\ \frac{1}{c_{cap,c}} \left[ -\phi_{phot,c} + c_{resp,c} x_1 2^{(0.1X_T-2.5)} + u_2 \right] \end{bmatrix}, \ x = \begin{bmatrix} X_d \\ X_c \end{bmatrix}, \ u = \begin{bmatrix} U_{led} \\ U_c \end{bmatrix} \\ t_0 &= 0, \ t_f = 21 \times 24 \times 60 \times 60, \ x(t_0) = \begin{bmatrix} 1.36 \cdot 10^{-3} \\ 9.82 \cdot 10^{-4} \end{bmatrix} \\ 0 &\leq x_2(t) \leq X_{Cmax}, \ 0 \leq u_1(t) \leq U_{ledmax}, \end{split}$$

(Yang, 2019),  $c_{cost}$  is  $4.20 \cdot 10^{-5}$  RMB m<sup>-2</sup> s<sup>-1</sup>. Equation (12) thus represents the influence of the lettuce price and the electricity price on profitability.

Our second optimal control problem formulation is represented by,

$$J_{2} = c_{lettuce}c_{fw}x_{1}(t_{f}) - \int_{t_{0}}^{9} \left(\frac{c_{energy}}{c_{unit}c_{trans}}u_{1}(t) + c_{co2}u_{2}(t) + c_{cost}\right)dt$$
(14)

The initial value  $x_2(t_0) = 9.82 \cdot 10^{-4} \text{ kg}[\text{CO}_2] \text{ m}^{-3}$  corresponds with 500 ppm. Finally in Equation (11), Equation (13), the plant factory temperature  $X_T$  is taken to be the setpoint of 24 °C. In Equation (11), the plant factory CO<sub>2</sub> concentration  $X_c$  is taken to be the setpoint of 1000 ppm as shown in Table 1, while in Equation (13) it is regarded as a second state variable.

A fresh lettuce price  $c_{lettuce}$  of 34.5 RMB kg[fw]<sup>-1</sup> in a plant factory is obtained from JD.com (https://item.jd.com/ 100001707609.html, a Chinese e-commerce platform). The average open field lettuce price over the whole year 2019 shown in xinfadi.com (http://www.xinfadi.com.cn/ as marketanalysis/0/list/1.shtml?prodname=%E6%95%A3%E7% 94%9F%E8%8F%9C&begintime=2019-01-01&endtime=2019-12-31, a Beijing vegetable data website) is 5.01 RMB kg[fw]<sup>-1</sup>. This much lower lettuce price than that from JD.com occurs because, compared to lettuce produced in a plant factory, the lettuce produced in the open field is dirtier and of much reduced quality. Also, the costs to grow such lettuce are much lower. The reason for investigating optimal control of a plant factory while taking account of the open field lettuce price is the following. The lettuce produced in a plant factory may not be sold completely in China, for the high price of 34.5 RMB kg [fw]<sup>-1</sup>. Then the remaining lettuce must be sold for a lower price. According to bj.bendibao.com (http://bj.bendibao.com/ zffw/201374/109201.shtm, a data website for Beijing), the agricultural electricity price cenergy in Beijing varies in between 0.30 and 0.92 RMB kWh[E]<sup>-1</sup>.

With the LED light intensity of 200  $\mu$ mol m<sup>-2</sup> s<sup>-1</sup> (red/blue ratio = 4:1), 44.56 W[PAR] m<sup>-2</sup> radiation requires 73.53 W[E] m<sup>-2</sup> electric power. So, the transfer efficiency from electricity to light radiation  $c_{trans}$  is 60.6%. To represent the light intensity in the commonly used unit of  $\mu$ mol m<sup>-2</sup> s<sup>-1</sup> instead of W[PAR] m<sup>-2</sup>, one must multiply with 4.49 in the current experimental setup. Because the upper limits of red and blue LED light intensity are 500  $\mu$ mol m<sup>-2</sup> s<sup>-1</sup> and 360  $\mu$ mol m<sup>-2</sup> s<sup>-1</sup> respectively, the total LED light intensity has an upper bound  $U_{ledmax}$ of 140 W[PAR] m<sup>-2</sup>. According to Van Henten and Bontsema (2009), the upper bound of CO<sub>2</sub> concentration  $X_{cmax}$  is 1400 ppm while the upper bound of the CO<sub>2</sub> supply rate  $U_{cmax}$  is 1.2 · 10<sup>-6</sup> kg[CO<sub>2</sub>] m<sup>-2</sup> s<sup>-1</sup>. The price  $c_{co2}$  of CO<sub>2</sub> supply is specified to be 4 RMB kg[CO<sub>2</sub>]<sup>-1</sup> as obtained from a market analysis.

#### 2.5. Optimal control algorithms

The "lightning fast" optimal control toolbox PROPT, which is part of TOMLAB, a set of optimisation tools for MATLAB, is used to solve the optimal control problems very efficiently. The high efficiency of the PROPT algorithm relates to pseudospectral collocation and polynomial approximations that are employed (Rutquist & Edvall, 2010). The optimal continuoustime control signals are approximated by polynomials. Plant factories are controlled by digital computer control systems generating piecewise constant controls. Therefore, to compute optimal controls for a plant factory, a digital optimal control algorithm must be employed. This algorithm is described in Van Straten et al. (2011) and uses what is generally called a piecewise constant control parametrisation. But here it represents the true digital piecewise constant control. The piecewise constant control turns the optimal control problem into a nonlinear programming problem solved by the MATLAB function 'fmincon'. This function is supplied with gradients that are computed very efficiently using the costate equations of digital optimal control. The efficiency of this digital optimal control computation is further improved by feeding it with a highly accurate initial digital optimal control guess.

This guess is obtained by averaging the continuous optimal control obtained from PROPT over each sampling period  $T_s$ , which equals 10 min. The flow chart of this digital optimal control computation is shown in Fig. 2.

## 3. Results and discussion

#### 3.1. Calibration

The Nomenclature shows that six model parameters are calibrated. Their calibration is described in this section. Nine sets of experimental data, recorded under different environmental conditions, mentioned in Table 1, are used for calibration and validation. As shown in Table 3, from the data in each single set, all 6 parameters were calibrated.

Using the data from a single set, first,  $c_{fw}$ ,  $c_{\tau}$ ,  $c_{lar,d}$  and  $c_{pl,d}$ were calibrated as follows. Parameter  $c_{fw}$ , being fresh mass to dry mass ratio, is assumed to be constant and calibrated to be the sum of all fresh mass measurements divided by the sum of all dry mass measurements of the set. Similarly, parameter  $c_{\tau}$ being the root dry mass to lettuce dry mass ratio, is assumed to be constant and calibrated to be the sum of all root dry mass measurements divided by the sum of all leaf plus root dry mass measurements in the set. In a similar manner,  $c_{lar,d}$  being shoot leaf area ratio, is assumed to be constant and calibrated to be the sum of all leaf area measurements divided by the sum of all leaf dry mass measurements in the set.

According to Van Henten (1994),  $c_{pl,d}$  is given by,

$$\mathbf{c}_{pl,d} = \mathbf{c}_k \mathbf{c}_{lar,d} (1 - \mathbf{c}_\tau) \tag{15}$$

where  $c_k$  is the extinction coefficient of the canopy,  $c_{lar,d}$  is the shoot leaf area ratio and  $c_{\tau}$  is the root dry mass lettuce dry mass ratio. By substituting the calibrations of  $c_{\tau}$  and  $c_{lar,d}$ , Equation (15) provides the calibrated value of  $c_{pl,d}$ .

Next, the calibrated values of  $c_{fw}$ ,  $c_{\tau}$ ,  $c_{lar,d}$  and  $c_{pl,d}$  are substituted in the model. Each of the sets is then used by nonlinear least squares to simultaneously calibrate both  $c_{\alpha\beta}$  and  $c_{led,phot}$  using the measurements of dry mass  $X_d$  from the data set as follows. At day 1, 7, 14 and 21, recorded in Table 2, we use a single value of  $X_d$  being the average of the 4 or 8 measurements performed that day. The average is meant to represent the average crop, as described by the crop model.

A prerequisite to properly estimate  $c_{\alpha\beta}$  and  $c_{led,phot}$  together using nonlinear least squares, is that both parameters are



Fig. 2 – Chart of the digital optimal control computation. The star superscript denotes optimality, the overbar denotes average.

Table 3 – Calibrations obtained from single data sets.									
Parameter \ Data set	1	2	3	4	5	6	7	8	9
C <sub>fw</sub>	18.22	18.78	16.70	22.82	20.71	18.64	24.74	26.24	19.61
C <sub>τ</sub>	0.071	0.089	0.083	0.100	0.085	0.086	0.078	0.089	0.077
C <sub>lar,d</sub>	39.21	32.02	25.44	41.71	30.37	23.39	44.58	42.38	28.56
C <sub>pl,d</sub>	32.79	26.26	20.99	33.81	25.00	19.25	36.98	34.76	23.72
$C_{\alpha\beta}$	0.52	0.51	0.46	0.51	0.53	0.42	0.68	0.41	0.44
$c_{led,phot}  imes 10^{-9}$	12.47	11.94	12.80	11.36	11.75	12.49	11.94	12.19	12.06
MRE for calibrated in %	46.26	25.59	29.13	15.40	38.54	7.84	19.62	27.70	23.02
MRE = mean relative error.									

identifiable from measurements of dry mass X<sub>d</sub>. This guarantees that the parameters can be uniquely determined from these measurements if the measurements are without error. Identifiability is investigated by means of a highly efficient algorithm (Stigter & Molenaar, 2015; Xu et al., 2018a) that computes a sensitivity matrix. Theoretically, if the  $n_{\nu}$  (number of parameters to be estimated which is two in our case) singular values of the sensitivity matrix are all non-zero, identifiability holds. Because the algorithm is of a numerical nature, zero means being in the order of the machine constant of  $2.2 \times$ 10<sup>-16</sup> or showing a clear gap between non-zero and "very small" singular values. The sensitivity matrix is computed from a model trajectory, in our case the one shown in Fig. 2, having parameters values as mentioned in the Nomenclature. The two singular values obtained from the algorithm are: 7.23,  $8.18 \times 10^{-5}$ . The gap between these two singular values is of the order  $10^5$  which is considered rather large (Stigter & Molenaar, 2015). This implies that simultaneous estimation of  $c_{\alpha\beta}$  and  $c_{led,phot}$  from measurements of dry mass  $X_d$  may be inaccurate. Nevertheless, their calibrations listed in Table 3 are in the expected range. This is satisfactory for our application, which is to produce reasonable estimates of lettuce dry mass  $X_d$ .

Table 3 shows significant variations, depending on which data set is used for calibration. In general, this is caused by both measurement and modelling errors. As to the measurements, significant variations occur, as shown in Fig. 3 where the red open dots indicate dry mass measurements of individual crops. These variations are mostly larger than the errors generated by the equipment and procedure to measure masses. Our model describes the growth of a single crop, where the underlying assumption is that either all crops are (approximately) identical or the crop reflects the average crop. Both assumptions are violated in practice. Together, this implies that modelling errors dominate measurement errors. In addition, the crop growth model Equation (1), Equation (4) is rather simple, one reason being that the dependence of photosynthesis on temperature is empirically modelled using a polynomial.



Fig. 3 – Modelled lettuce growth versus measured lettuce growth recorded in dataset 6 that is used for validation. For calibration and validation, we used the average value of the measurements of lettuce dry mass at day 1, 7, 14 and 21.

# 3.2. Validation

To further analyse the accuracy of the calibrated model, it is good common practice to validate the model using a data set that has not been used for calibration, also referred to as an independent data set. If, out of the 9 sets we obtained, we use one for validation, this can be done in 9 different ways. The results are shown in Table 4 in which the set used for validation is indicated by the corresponding number on top of each column. The calibrated parameters in the corresponding column are obtained from the remaining 8 data sets.

Taking the mean relative error of the validation as a measure of accuracy, the column that uses set 6 for validation provides the best outcome and is therefore typed bold. The calibrations mentioned in this column, are taken as the final calibrations of the model parameters, mentioned in the nomenclature. The corresponding MRE of 7.84%, mentioned in Table 4, is relatively low and justifies analysis by means of optimal control.

From the last two rows of Table 4, observe that accuracy has improved a lot as compared to the uncalibrated model. On the other hand, from the rather large variations in Table 3 as well as the MRE values of the validation in Table 4, one must conclude that the calibration and validation are not highly accurate.

We already commented that this is mostly due to simplifying assumptions underlying the crop growth model Equation (1), Equation (4). Another simplifying assumption underlies our calibration. It assumes the six calibrated parameters to be independent of environmental conditions. Finally, recall the rather large gap between the two singular values from the identification analysis, presented in this section. These reveal that the simultaneous calibration of  $c_{\alpha\beta}$  and  $c_{led,phot}$  may not be very accurate, although they are in the correct range, sufficient to produce reasonable estimates of lettuce dry mass  $X_d$ .

Figure 3 shows the match between the lettuce dry mass computed from the calibrated model and the measurements of lettuce dry mass taken from validation data set 6. The corresponding standard deviations are given in Table 5. These apply to the average of the lettuce measurements at day 1, 7, 14 and 21 shown in Fig. 3.

The  $c_{pl,d}$  value obtained by Van Henten (2003) relied on calibration with the varieties of 'Berlo' and 'Norden' under greenhouse conditions (Van Henten, 1994). Apart from model deficiencies that we discussed, the possibly thinner leaves in our lettuce variety, and the morphology change caused by LED light replacing solar light, might account for the decrease of  $c_{pl,d}$  compared to Van Henten (2003). Note that  $c_{led,phot}$  is also smaller than  $\varepsilon$  from Van Henten (1994).

Apart from model deficiencies and different growing conditions and lettuce varieties, this may also be explained from research showing that Sunlike-LEDs (Zou et al., 2020) or LEDs with more light wavelengths than simply red and blue (Li et al., 2020) lead to greater lettuce dry mass. Besides, the prolonged photoperiod might also be the reason for the decrease of light use efficiency (Zha et al., 2019).

# 3.3. Maximum profits for sales against a plant factory lettuce price

Taking the plant factory lettuce price of 34.5 RMB  $kg[fw]^{-1}$  from JD.com and the agricultural electricity price within

Table 5 – Standard deviations obtained during validation against data set 6, see also Fig. 3.						
Measurement of lettuce dry mass at day	Standard deviation					
1	$1.60 \cdot 10^{-4}$					
7	$9.56 \cdot 10^{-4}$					
14	$6.31 \cdot 10^{-3}$					
21	$1.29 \cdot 10^{-2}$					

Table 4 — Model validations using single data sets. The single data set used for validation is indicated by its number 1 to 9. The calibrated parameter values in the corresponding column are equal to the mean value of the calibrations in Table 3 as obtained from the other 8 data sets. MRE indicates the mean relative error.

Calibrated parameter $\smallsetminus$ Data set	1	2	3	4	5	6	7	8	9
c <sub>fw</sub>	21.03	20.96	21.22	20.46	20.72	20.98	20.22	20.03	20.86
$C_{\tau}$	0.086	0.084	0.084	0.082	0.084	0.084	0.085	0.084	0.085
C <sub>lar,d</sub>	33.56	34.46	35.28	33.24	34.66	35.53	32.89	33.16	34.89
$c_{pl,d}$	27.60	28.41	29.07	27.47	28.57	29.29	27.07	27.35	28.73
$C_{lphaeta}$	0.50	0.50	0.50	0.50	0.49	0.51	0.48	0.51	0.51
C <sub>led,phot</sub>	12.07	12.13	12.03	12.21	12.16	12.06	12.13	12.10	12.12
MRE in %	46.26	25.59	29.13	15.40	38.54	7.84	19.62	27.70	23.02
MRE for uncalibrated model in %	357.75	519.64	498.44	212.55	316.89	432.43	71.33	233.99	295.56



Fig. 4 – Optimal control figures for sales against the plant factory lettuce price.

0.30-0.92 RMB kWh[E]<sup>-1</sup> from Beijing, the optimal control problem represented by Equation (11), Equation (12) is solved. The optimal lettuce growth, LED light intensity, and the costate of the lettuce dry mass are shown in Fig. 4.

The LED light intensity control pattern shows that if the harvested lettuce can be sold at the price shown in JD.com for a plant factory, optimal control directs the LED light intensity to the upper bound of the plant factory during light periods. Recall that dark periods between 22:00 and 06:00 are enforced by switching the upper bound U<sub>ledmax</sub> of U<sub>led</sub> to zero. This causes the switches of LED light intensity U<sub>led</sub> in Fig. 4. Note that as the lettuce grows bigger, the lettuce growth rate that relates to both gross photosynthesis rate  $c_{\alpha\beta}\varphi_{phot,c}$  and gross respiration rate  $c_{resp.d}X_d 2^{(0.1X_T-2.5)}$ , as given in Equation (1), becomes almost constant. Further results are summarised in Table 6. Fresh lettuce harvest is represented by  $c_{fw}X_d(t_f)$  in Equation (12). The optimal control is almost bang-bang indicating that crop revenue is sufficient to justify full artificial lighting. In the next section, where crop revenues are assumed much smaller, the optimal control input is no longer bang-bang everywhere, as shown in Fig. 5.

# 3.4. Maximum profits for sales against an open field lettuce price

Due to its high price of 34.5 RMB kg[fw]<sup>-1</sup> obtained from JD. com, lettuce produced by the plant factory may not be completely sold. Then the remaining lettuce must be sold against a low price, such as the open field lettuce price of 5.01 RMB kg[fw]<sup>-1</sup> obtained from xinfadi.com. Maximum profits obtained from optimal control computations for the low open field lettuce price of 5.01 RMB kg[fw]<sup>-1</sup> and the same electricity prices as those in Table 6 are recorded in Table 7.

The revenues of selling crops minus the energy cost to produce them all come out positive. However, due to the fixed  $t_f$ 

cost over 21 days 
$$\int c_{cost} dt =$$
 76.19 RMB m<sup>-2</sup> in Equation (12),

maximum profits all come out negative. This indicates that plant factory production of lettuce is not profitable when all lettuce must be sold against the low open field lettuce price of 5.01 RMB kg[fw]<sup>-1</sup>.

Table 6 – Optimal control results with the plant factory fresh lettuce price of 34.5 RMB kg[fw] <sup><math>-1</math></sup> .								
Electricity price (RMB kWh[E] <sup>-1</sup> )	Fresh lettuce harvest (kg[fw] m <sup>-2</sup> )	Revenues (RMB m <sup>-2</sup> )	Energy (kWh[E] m <sup>-2</sup> )	Energy cost (RMB m <sup>-2</sup> )	Profit (RMB m <sup>-2</sup> )			
0.30	10.57	364.60	78.43	23.53	264.88			
0.61	10.57	364.60	78.43	39.22	240.56			
0.92	10.57	364.60	78.43	72.16	216.25			



Fig. 5 – Optimal control figures for sales against the open field lettuce price.

Table 7 – Optimal control results with open field fresh lettuce price of 5.01 RMB kg[fw] <sup><math>-1</math></sup> .								
Electricity price (RMB kWh[E] <sup>-1</sup> )	Fresh lettuce harvest (kg[fw] m <sup>-2</sup> )	Revenues (RMB $m^{-2}$ )	Energy (kWh[E] m <sup>-2</sup> )	Energy cost (RMB m <sup>-2</sup> )	Profit (RMB m <sup>-2</sup> )			
0.30	10.57	52.95	78.43	23.53	-46.77			
0.61	8.89	44.52	62.82	38.32	-69.99			
0.92	0.018	0.091	0	0	-76.10			

Table 8 – Profit against the lettuce price of 5.01 RMB kg[fw] $^{-1}$ under different LED light intensities.							
Electricity price (RMB kWh[E] <sup>-1</sup> )	Maximum profit (RMB m <sup>-2</sup> )	Profit with 140 W[PAR] $m^{-2}$ (RMB $m^{-2}$ )	Profit with 44.56 W[PAR] $m^{-2}$ (RMB $m^{-2}$ )				
0.30	-46.77	-46.89	-68.05				
0.61	-69.99	-70.95	-75.71				
0.92	-76.10	-95.02	-83.37				

The profit obtained with a constant light intensity amplitude of 140 W[PAR] m<sup>-2</sup>, which is equal to the upper bound  $U_{ledmax}$  used for optimal control, and electricity prices of 0.30, 0.61 and 0.92 RMB kWh[E]<sup>-1</sup>, is shown in Table 8. For the electricity prices 0.30 and 0.61 profit comes close to the maximum profit, copied from Table 7 into Table 8. This happens because the optimal control is at its upper bound most of the time, like it is in Fig. 3 and Fig. 4. With a reduced constant light intensity amplitude of 44.56 W[PAR] m<sup>-2</sup>, that corresponds with 200 µmol m<sup>-2</sup> s<sup>-1</sup> which is commonly used in the experimental setup, and the same electricity prices, profit reduces significantly, as seen from Table 8.

When the electricity price is 0.92 RMB  $kWh[E]^{-1}$ , optimal control reveals that it is not advisable to supply LED lighting. This can also be concluded from Table 7. One should

note that the positive value of revenues with the electricity price being 0.92 RMB kWh $[E]^{-1}$  is caused by the initial lettuce dry mass as a pre-production investment. Discarding the negative profit, supply of LED lighting no longer increases profit when the electricity price becomes higher than 0.84 RMB kWh $[E]^{-1}$ . This result is not shown in any Table.

When the electricity price is 0.61 RMB kWh $[E]^{-1}$ , optimal LED light intensity increases in the first few days, then it reaches the upper bound of LED light intensity as shown in Fig. 5. This is because at an early age the plant is much smaller, demanding lower light intensity for proper lettuce growth. This complies with Ioslovich (2009) who also found that lettuce requires a higher light intensity as it grows bigger. The costate of the lettuce dry mass is also shown in Fig. 5. It is



Fig. 6 – Surface plot of profit (vertical) against electricity price and lettuce price.

much lower compared with that shown in Fig. 4 because of the low lettuce price.

# 3.5. Maximum profit as a continuous function of the lettuce and electricity price

Thanks to the high efficiency of optimal control computations realised by the PROPT toolbox of TOMLAB and the fact that plant factory analysis can be performed off-line, one can easily compute a large series of optimal controls for varying electricity price and lettuce price. Doing so one can produce a surface plot directly showing their influence on profit. Figure 6 presents such a surface plot having 441 grid points being the outcome of 441 optimal control computations.



Fig. 7 – States, control inputs, and costates of co-optimising LED lighting and CO<sub>2</sub> supply.

Electricity price and lettuce price vary along the bottom two axes whereas profit is represented on the vertical axis. This 3D plot gives a direct and global overview. Overviews, like the one in Fig. 6, are most interesting to growers, legislators, and governments in promoting plant factories because they directly show the influence of changes and choices (in this case of the lettuce and electricity price) on maximum profit. Since optimal control puts no limits on introducing more advanced models as well as additional controls and control objectives, the influence of these can be directly visualised in a similar manner.

Figure 6 shows a rather simple, straightforward, almost flat surface. From this surface, one can immediately conclude that maximum profit is almost everywhere linearly decreasing with the electricity price and linearly increasing with the lettuce price. This is because the optimal control of LED light intensity is at its upper bound of 140 W m<sup>-2</sup> most of the time. The red surface in Fig. 6 represents zero profit. From its intersection with the surface plot, we conclude that profit is only positive when the lettuce price is over 20 RMB kg[fw]<sup>-1</sup>.

### 3.6. Co-optimising LED lighting and CO<sub>2</sub> supply

If plant factory climate dynamics and their control are also included, the situation becomes more realistic but also more complicated. Recall the interesting feature of the optimal control approach to handle all such complications. To illustrate this, an example of co-optimising LED lighting and  $CO_2$  supply, is presented in this section. The optimal control problem formulation is given by Equation (13), Equation (14).

The optimal control inputs, states and costates are shown in Fig. 7. During light periods, the  $CO_2$  supply causes the  $CO_2$  to stay at its upper bound which is most beneficial for growth. As the lettuce grows bigger it releases more  $CO_2$  during dark periods through respiration. The negative spikes in  $CO_2$  concentration pointing downwards, occur before dark periods. They leave room for  $CO_2$  to not exceed its upper bound through respiration during dark periods. Observe that whenever the costate of the  $CO_2$  concentration is negative, the increase of  $CO_2$  concentration brings harm to profit. This happens especially during dark periods at a late age.

With the electricity price of 0.61 RMB kWh[E]<sup>-1</sup> and lettuce price of 5.01 RMB kg[fw]<sup>-1</sup>, the profit is -69.35 RMB m<sup>-2</sup> and the fresh lettuce harvest is 10.00 kg[fw] m<sup>-2</sup>. They are both slightly higher than that in Table 7 because of the increase in CO<sub>2</sub> concentration. With the electricity price of 0.61 RMB kWh[E]<sup>-1</sup> and lettuce price of 34.5 RMB kg[fw]<sup>-1</sup>, the profit is 254.61 RMB m<sup>-2</sup> and the fresh lettuce harvest is 11.01 kg[fw] m<sup>-2</sup>.

# 4. Conclusions

Optimal control is a highly suitable tool to investigate the profitability of systems because it puts few constraints on the type of system and the number and types of controls, control objectives and constraints. In this paper, it was used to investigate the profitability of a Chinese plant factory growing lettuce. To that end, a lettuce growth model was modified from one that applies in a greenhouse to one that fits a plant factory. Having 9 sets of data, each containing 100 crop measurements performed in a plant factory, we used 8 sets for calibration of 6 model parameters and 1 set for validation. The mean relative errors obtained from 9 different validation sets varied from 7.84% to 46.26% revealing that the crop model is not highly accurate. The crop measurements themselves showed variations up to 100%. This reveals that results obtained from a single crop growth model applied to an entire plant factory should at best be interpreted as an "average crop".

Taking maximum profit as the optimal control objective and using the calibrated crop model with the smallest mean relative validation error of 7.84%, the following results concerning profitability were obtained. When lettuce can be sold as a plant factory product as shown in the e-commerce platform, profit was up to 264.88 RMB m<sup>-2</sup> with the agricultural electricity price of 0.3 RMB kWh[E]<sup>-1</sup> and a photoperiod of 16 h per day. The optimal LED light intensity pattern reached its upper bound every day during light periods, even for the highest agricultural electricity price of 0.92 RMB kWh [E]<sup>-1</sup>.

When lettuce has to be sold as an open field product, profit is negative, discouraging growers to start plant factory production. Discarding this negative profit, LED lighting in plant factories is advisable when electricity prices are lower than  $0.84 \text{ RMB kWh}[E]^{-1}$ . Profit becomes positive only when the lettuce price is over 20 RMB kg[fw]<sup>-1</sup>.

Optimal control patterns reveal that if the setpoint of conventional LED light controllers would be set to its maximum value, profit comes close to its maximum for electricity prices below 0.61 RMB kWh[E]<sup>-1</sup>, for a fixed dark time of eight hours a day. The reason is that, during the photoperiod of 16 h, optimal control is at its upper bound most of the time. Selecting a lower setpoint, which is common practice, significantly reduces profit. Obviously, the profit computed from optimal control can serve as a prediction to growers before starting actual production. Moreover, the optimal control of LED light intensity can also be supplied to the plant factory in real-time because it realises the lettuce's physiological demand for a fixed dark period, which we took to be eight hours a day.

Co-optimising LED lighting and  $CO_2$  supply was also performed to show that optimal control can involve environmental factors of the plant factory once properly modelled. In addition to this, just as the second control objective in this paper was extended with cost of  $CO_2$  supply, maximisation of profit can be replaced or combined with other criteria such as improving sustainability.

Improving the quality of dynamic models describing crop growth and environmental conditions, as well as improving or manipulating control objectives to incorporate sustainability, are challenging topics for future research concerning plant factory production. Compared to greenhouses, plant factories are favourable from the perspective of modelling and control, because they are less influenced by uncertain, ill-predictable, external conditions, such as the weather. This is expected to enhance the benefits that can be obtained from implementing optimal control and improvements in modelling the system dynamics.

### **Declaration of competing interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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