NEW PERSPECTIVES FOR OPTIMAL CONTROL OF DRYING PROCESSES

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Abstract: A new distributed model that describes the drying of individual particles is presented. It considers internal resistances towards mass transfer, controlled by diffusion and sorption, as well as external resistances. The parameters in this model all have a clear physical meaning and interpretation. Using a measure of the product quality, which deteriorates during drying, and a measure for the energy costs of drying, optimal control trajectories are computed for different drying times. The computations are based on a finite-difference approximation of the infinite-dimensional model. The results indicate the feasibility of the model to optimize drying processes of different type as well as the significance of optimizing the drying time. *Copyright © 2000 IFAC*

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1. INTRODUCTION

The most important aspects of optimization of drying processes are energy consumption and the product quality, which deteriorates during drying (Banga and Singh, 1994). Since the quality is characterized by the spatial distribution of moisture content and temperature inside the product, it requires accurate modeling of the heat and mass transfer at the level of individual particles (material level) during drying. The moisture content, temperature and quality differ significantly inside each particle (Quirijns *et al.*, 1998), requiring a model of distributed nature. Many models of drying processes have been presented in the literature ranging from empirical and semi-empirical to almost purely physical models (Fasina and Sokhansanj, 1996; Kerkhof, 1994; McMinn and Magee, 1996; Kiranoudis *et al.*, 1993). The drawback of the first two types of models is that it is difficult to extend them to different products and process conditions because their parameters depend on both in an unknown manner. The drawback of models of the latter type is that they are often too complicated for computation. Based on a number of assumptions that usually hold in practice, a distributed model is presented which describes the main transport properties inside and outside each particle during drying.

In biological material, the controlling resistance for drying is the internal mass transfer resistance, governed by moisture diffusion. Therefore the diffusion coefficient is a major parameter in drying models, which is a "lumped" parameter, involving all internal physical phenomena, known as well as unknown mechanisms. Since it is not possible to theoretically predict this parameter based on composition or structure of the material, generally *empirical* relations are chosen and fitted to obtain the so called effective diffusion coefficient (Zogzas *et al.*, 1996; McMinn and Magee, 1996; Waananen *et al.*, 1993). Often these relations state that the effective diffusion coefficient depends on moisture content, representing the decreasing drying rate as moisture content decreases. The effective diffusion coefficient has no clear physical meaning and furthermore it is not likely that such empirical relations cover the whole range of experimental conditions.

To maximally benefit from optimal control computations an accurate model and cost function must be selected for the whole range of drying conditions. To this end a new distributed model is developed, in which it is assumed that the decrease of drying rate with respect to moisture content is a result of the conversion from bound to free water inside the particle. Kiranoudis *et al.* (1995) modeled this conversion by a reversible reaction. In fact

however, this conversion should be physically described by a sorption process, which takes place at low moisture contents (section 2). This description allows the use of the "true" diffusion coefficient in the model, which now only depends on temperature. Moreover, the parameters describing the sorption process can be derived from the sorption isotherm. Hence, in the model obtained in this manner, the parameters have a physical meaning and are known or can be estimated in a known manner. The choice of this model is further motivated and explained in Quirijns (1998).

Based on a finite-difference approximation of the distributed model and a measure of product quality and energy consumption (section 3), optimal control trajectories are computed (section 4), using control parameterization (van Willigenburg and Loop, 1991) for different drying times. The results indicate the feasibility of the model to optimize drying processes of different type by increasing the quality and saving the energy costs. In addition the results indicate the significance of optimizing the drying time.

2. THE MODEL

The moisture transfer in each particle is modeled by three basic transfer phenomena: free water diffusion, conversion between bound and free water inside the particle and external convection at the boundary of the particle. The state variables of the infinite-dimensional model are bound and free moisture content, X_B and X_F respectively, and temperature T_m. Appendix 1 presents the full nomenclature and the parameter values used for optimization.

Conversion between bound and free water
The interaction between water and the solid is modeled through the adsorption of free water to become bound water and the desorption of bound water to become free water. The model, which describes these sorption processes is analogous to the model of heterogeneous catalysis and relies on the following assumptions (Aris, 1975; Quirijns, 1998):
a) the solid surface is covered with a number of sites at which water molecules may be held.
b) the number of active sites per unit area, X_m is constant.
c) all sites are equivalent.
d) each sorbed molecule occupies only one site.
e) there is no interaction between adsorbed molecules.
f) the thermodynamic variables for sorption are derived from a desorption isotherm.
g) during drying, moisture content, vapor pressure and temperature are in thermodynamic equilibrium within the drying material and with the drying environment.

The rate of adsorption of free water is proportional to the product of the concentration of those sites not yet occupied and the concentration of free water above the surface,

$$r_a = k_a \cdot X_F \cdot X_m \cdot (1 - \theta) \qquad (1)$$

where

$$\theta = \frac{X_B}{X_m} \qquad (2)$$

From *(1)*, *(2)*,

$$r_a = k_a \cdot X_F \cdot (X_m - X_B) \qquad (3)$$

On the other hand the rate of desorption is proportional to the concentration of sites already occupied,

$$r_d = k_d X_m \theta \qquad (4)$$

From *(4)*, *(2)*,

$$r_d = k_d X_B \qquad (5)$$

The desorption rate *(5)* minus the adsorption rate *(3)* equals the decrease of bound water and the increase of free water inside the particle,

$$-\frac{dX_B}{dt} = \frac{dX_F}{dt} = k_d X_B - k_a X_F (X_m - X_B) \qquad (6)$$

At *equilibrium* the adsorption rate *(3)* and the desorption rate *(5)* balance resulting in:

$$k_d X_B = k_a X_F (X_m - X_B) \qquad (7)$$

From *(7)* at equilibrium the ratio k_a/k_d, which is called an equilibrium constant K_e, satisfies:

$$K_e = \frac{k_a}{k_d} = \frac{X_{B,e}}{X_{F,e}(X_m - X_{B,e})} \qquad (8)$$

where the index *e* refers to equilibrium.

Partial differential equations, boundary and initial conditions of the drying model
The following partial differential equations describe moisture diffusion, conversion from bound and free water and the thermodynamics inside the particle:

$$\frac{\partial(\rho_m X_F)}{\partial t} = \nabla(\rho_m D \nabla X_F) + \rho_m r_B \qquad (9.1)$$

$$-\frac{\partial X_B}{\partial t} = r_B \qquad (9.2)$$

$$\frac{\partial(\rho_m h_m)}{\partial t} = \nabla(\lambda_m \nabla T_m) \qquad (9.3)$$

with

$$r_B = k_d X_B - k_a (X_m - X_B) X_F \qquad (10.1)$$

$$h_m = c_{p,dm} T_m + X c_{pw} T_m \qquad (10.2)$$

$$X = X_F + X_B \qquad (10.3)$$

The first term after the equality sign in equation *(9.1)* represents free water diffusion inside the particle governed by the diffusion coefficient D. The second term together with equations *(9.2)* and

(10.1) describes the conversion between bound and free water inside the particle governed by the bound to free water conversion rate r_B. Finally equation (9.3) describes the enthalpy dynamics inside the particle governed by the thermal conductivity λ_m. From now on the particle will be considered cylindrical with a length far greater than its radius R. Then equations (9.1)-(9.3) turn into:

$$\frac{\partial X_F}{\partial t} = \frac{1}{r}\left[D\frac{\partial X_F}{\partial r} + r\frac{\partial D}{\partial r}\frac{\partial X_F}{\partial r} + rD\frac{\partial^2 X_F}{\partial r^2}\right] \quad (11.1)$$

$$-\frac{\partial X_B}{\partial t} = r_B \quad (11.2)$$

$$\frac{\partial T_m}{\partial t} = \frac{1}{r}\left[\lambda_X\frac{\partial T_m}{\partial r} + r\frac{\partial \lambda_X}{\partial r}\frac{\partial T_m}{\partial r} + r\lambda_X\frac{\partial^2 T_m}{\partial r^2}\right] \quad (11.3)$$

where

$$\lambda_X = \frac{\lambda_m}{\left(c_{p,dm} + c_{pw}X\right)\rho_m} \quad (12)$$

and where r denotes the distance measured from the center line of the cylinder which varies from 0 to R. Because of the symmetry at the center line of the cylindrical particle the following boundary conditions apply:

$$\left.\frac{\partial X_F}{\partial r}\right|_{r=0} = 0, t > 0 \qquad \left.\frac{\partial T_m}{\partial r}\right|_{r=0} = 0, t > 0 \quad (13)$$

At the boundary of the cylinder where $r=R$, external convection and heat transfer determine the following boundary conditions:

$$J_{XS} = -D\rho_m\left.\frac{\partial X_F}{\partial r}\right|_{r=R} = k\left(Y_{as} - Y_a\right)\rho_a \quad (14.1)$$

$$J_T = -\lambda_m\left.\frac{\partial T_m}{\partial r}\right|_{r=R} = h\left(T_m\big|_{r=R} - T_a\right) + \Delta H_v J_{XS} \quad (14.2)$$

Y_{as} is the absolute air humidity, which is in equilibrium with the moisture content at the surface of the material (assumption (g)). Y_{as} is determined by the water activity at the surface of the material and the saturated vapor pressure:

$$Y_{as} = \frac{M_w}{M_a}\frac{a_w P_{sat}}{\left(P_{tot} - a_w P_{sat}\right)} \quad (14.3)$$

where a_w is given by the GAB sorption isotherm (Lievense, 1991):

$$\frac{X}{X_m} = \frac{C_{aw}K_{aw}a_w}{\left(1 - K_{aw}a_w\right)\left(1 - K_{aw}a_w + C_{aw}K_{aw}a_w\right)} \quad (14.4)$$

and P_{sat} by the psychrometric relation:

$$P_{sat} = 1000 \cdot 10^{7.06262 - \left(1650.27/(T - 273.15 + 226.346)\right)} \quad (14.5)$$

At the start of drying, it is assumed that the moisture content is

$$X(r)\big|_{t=0} = 1 \text{ [kg}_{H2O}.\text{kg}^{-1}_{ds}] \quad (15.1)$$

and the material is assumed to be in equilibrium, so (8) must be satisfied. Solving (8) and (15.1) gives for $r \in [0, R]$:

$$X_F(r)\big|_{t=0} = 0.9337 \text{ [kg}_{H2O}.\text{kg}^{-1}_{ds}] \quad (15.2^1)$$

$$X_B(r)\big|_{t=0} = 0.0663 \text{ [kg}_{H2O}.\text{kg}^{-1}_{ds}] \quad (15.2^2)$$

Furthermore,

$$T_m(r)\big|_{t=0} = 298 \text{ [K]}, r \in [0, R] \quad (15.3)$$

while the remaining initial conditions are given by equation (16.3).

The sorption parameters X_m and K_e can be obtained from the sorption isotherm (14.4). X_m follows directly from the GAB equation, while K_e is related to the Gibbs free energy, which can easily be derived from the sorption isotherm. Since K_e is the ratio of k_a and k_d only one of these two parameters remains to be estimated. Since the other parameters in the model (11)-(14) all have a clear physical meaning and interpretation they are known in principal or can be estimated in a known manner (Quirijns, 1998).

3. PRODUCT QUALITY, ENERGY CONSUMPTION AND THE PERFORMANCE INDEX

In this section a description of product quality and energy consumption is presented in terms of the model in order to investigate the feasibility of the model to optimize drying processes. A performance index is obtained which represents the cash flow per unit of product per time unit obtained from drying. Let Q represent the residual activity of the biological material which is a measure of quality of for instance micro organisms and enzymes. The inactivation of micro organisms, among which *Lactobacillus plantarum* which is considered here, and enzymes can usually be described by first-order reaction kinetics. Then $Q(r)$ at each location r in the particle can be determined with the following differential equation (Lievense, 1991):

$$\frac{dQ(r)}{dt} = -k_i Q(r) \quad (16.1)$$

with

$$\ln(k_i) = \left[\left(a_1 - \frac{a_2}{\Re T_m}\right)X + \left(b_1 - \frac{b_2}{\Re T_m}\right)\right] \quad (16.2)$$

$$+ \left[1 - \exp\left(pX^q\right)\right]\left[\left(a_1' - \frac{a_2'}{\Re T_m}\right)X + \left(b_1' - \frac{b_2'}{\Re T_m}\right)\right]$$

and initial condition,

$$Q(r)\big|_{t=0} = 1, r \in [0, R] \quad (16.3)$$

Furthermore the energy costs to realize and maintain the trajectory of air conditions per unit product can be expressed as follows (van Boxtel and Knol, 1996):

$$J_D = \int_0^{t_f}\left(\gamma\rho_a\phi\left(c_{pa} + c_{pv}Y_a\right)\left(T_a - T_{env}\right)\right)dt \quad (17)$$

where 0 and t_f are the initial and final time of the drying process respectively, γ is the price of energy per unit product, ϕ is the air flow and T_a and Y_a are the temperature and humidity of the air in the dryer respectively. The latter three can be applied as control variables to improve the performance of the drying process. In the optimal control computations in this paper ϕ and Y_a will be fixed and T_a will be the only control variable. Let t_r denote the time necessary to prepare for the next drying run. Then from (16) and (17) the following performance index is obtained which equals the cash flow per unit of time and material obtained after the drying process:

$$J(T_a(t)) = \frac{1}{(t_r + t_f)} \left(\frac{\eta}{\pi R^2} \int_0^R 2\pi r Q(r)\big|_{t=t_f} dr - \right.$$

$$\left. \int_0^{t_f} \left(\gamma \rho_a \phi \left(c_{pa} + c_{pv} Y_a \right) (T_a - T_{env}) \right) dt \right) \qquad (18)$$

In equation (18) η represents the price obtained for a unit of dried material. The first spatial integral on the right in equation (18) is a function of the product quality at the final time. To compute the performance index the quality, which is governed by the differential equation (16), must be added as a state variable to the model. Then the spatial integral becomes a function of the final state while the second time integral in equation (18) represents the running costs.

4. OPTIMAL CONTROL COMPUTATIONS AND RESULTS.

A finite-difference approximation of the infinite dimensional model (11)-(14), (16) is used to compute the state evolution and performance, given the initial state and control. On the radius R of the cylinder N points are distinguished at equal distances,

$$r_i = (i-1)\Delta r, \ i = 1,2,..,N, \ \Delta r = \frac{R}{N-1} \qquad (19)$$

The first and second derivatives of X_F with respect to r are approximated for $i = 2,3,...,N-1$ with,

$$\frac{\partial X_F}{\partial r} = \frac{X_F(r_{i+1}) - X_F(r_{i-1})}{2\Delta r} \qquad (20.1)$$

$$\frac{\partial^2 X_F}{\partial r^2} = \frac{X_F(r_{i+1}) - 2X_F(r_i) + X_F(r_{i-1})}{\Delta r^2} \qquad (20.2)$$

and similarly for T_m and λ_X. Furthermore

$$\frac{\partial D}{\partial r} = \frac{\partial D}{\partial T_m} \frac{\partial T_m}{\partial r} \quad \text{and} \quad D = D_0 e^{\frac{-E_a}{RT_m}} \qquad (21a, 21b)$$

where the first term on the right is obtained in analytic form from equation (21b) while the second is approximated by (20.1) with X_F replaced by T_m. From (13) and (14), at the boundaries the first derivatives of X_F with respect to r are fixed and similarly for T_m. Then for the second derivative the following approximation was used for X_F at $r = R$:

$$\frac{\partial^2 X_F}{\partial r^2}\bigg|_{r=R} = \frac{\left(\frac{\partial X_F}{\partial r}\big|_{r=R} \Delta r - \left(X_F(r_N) - X_F(r_{N-1}) \right) \right)}{\Delta r^2 / 2} \qquad (22)$$

and similarly for T_m. Finally for λ_X at $r = R$ the approximation

$$\frac{\partial \lambda_X}{\partial r}\bigg|_{r=R} = \frac{\lambda_X(r_N) - \lambda_X(r_{N-1})}{\Delta r} \qquad (23)$$

was used. Given the spatial discretization scheme (19) the spatial integral in (18) was computed using the trapezoidal numerical integration rule. The time integration was performed with the Matlab/Simulink integrator ode15s for stiff systems. The number N was taken equal to 15 because larger values of N did not result in significant changes.

Although the performance index represents the cash flow obtained after drying it does not take into account the goal, which is to dry the product up to a certain level. Therefore the performance index is modified by a penalty function J_X, resulting in the performance index J_{DX} to be minimized:

$$J_{DX} = J_X - J \qquad (24.1)$$

where,

$$J_X = c_1(X - X_d), \ X > X_d \ , \ J_X = 0, \ X \leq X_d \qquad (24.2)$$

in which X_d represents the desired moisture concentration at the end of drying. The coefficient c_1 is used to weigh the goal to reach X_d against the cash flow during optimization.

For different fixed final times t_f the control interval $[0, t_f]$ was divided into N_u equidistant time-intervals over which the control variable remains constant. Then the piecewise constant control $T_a(t)$, $t \in [0, t_f]$ is uniquely determined by N_u values of the control variable. Then, given the initial state, the determination of the optimal control that minimizes (24) constitutes a function minimization where the variables are the N_u values of the control variable which is bounded from above and below:

$$313 \leq T_a \leq 360 \ [K] \qquad (25)$$

N_u equal to 1 corresponds with static control, where J_{DX} is minimized while the control variable is kept constant during the operation.

Figure 1 shows the results obtained from function minimization using different values of N_u and t_f and a final desired moisture content $X_d = 0.05$ (upper three) and $X_d = 0.075$ (lower three) [kg$_{H2O}$.kg$^{-1}_{ds}$]. As expected, increasing N_u results in better performance, except at some points, which is probably due to the existence of local minima. Furthermore figure 1 shows that the optimal final time is about 12000 [s] for $X_d = 0.05$ [kg$_{H2O}$.kg$^{-1}_{ds}$] and that the quality after drying is sensitive to the final desired moisture content. A higher value for X_d at the same t_f allows drying at more moderate temperatures, causing less inactivation of the thermal sensitive biological material and a decrease in energy costs. Figure 2 shows the optimal control
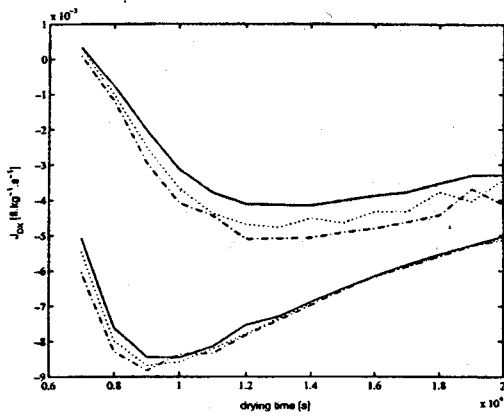
Fig. 1: Optimal performance as function of final drying time for $N_u = 1$:-, $N_u = 3$:—, $N_u = 6$: -.-. and $X_d = 0.05$ and 0.075 [kg$_{H2O}$.kg^{-1}$_{ds}$]

Fig. 2a: Optimal control trajectory for $t_f = 12000$ [s], $X_d = 0.05$ [kg$_{H2O}$.kg^{-1}$_{ds}$] and $N_u = 6$, $J_{DX} = -J_D = -5.1 \cdot 10^{-3}$ [fl.kg^{-1}.s^{-1}]
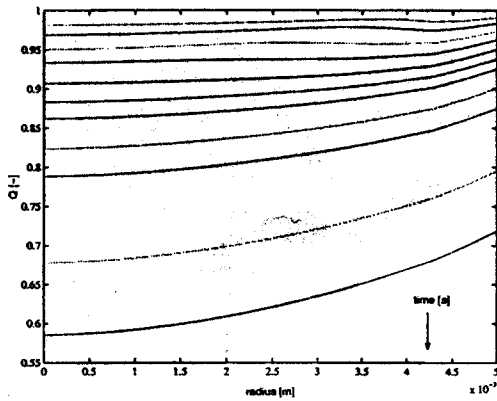
Fig. 2b: Optimal quality profiles for $t_f = 12000$ [s], $X_d = 0.05$ [kg$_{H2O}$.kg^{-1}$_{ds}$] and $N_u = 6$ (every 1000 [s])

trajectory, the average material temperature and quality profiles of the solution with $N_u = 6$ and $t_f = 12000$ [s]. The considered micro organism is less sensitive to temperature at lower moisture contents, which explains the trajectory of T_a. The initial high temperature does not damage the quality, since the

Fig. 3: Sensitivity to the control for $N_u = 1$, $t_f = 12000$ [s], $X_d = 0.05$ [kg$_{H2O}$.kg^{-1}$_{ds}$]

corresponding wet bulb temperature of the material in this period of the drying process is low. After this period the temperature of the material rises and T_a is decreased to protect the material during the temperature-sensitive phases. When during drying the moisture content is reduced the material is less sensitive to inactivation and T_a can increase to reach X_d. Compared to the static control with a constant T_a of 327 [K], this leads to a higher product quality and less energy costs and therefore to an improved value of the performance index (24%). Figure 3 illustrates the sensitivity of the two components of the performance index (24) to the value of the control variable if it remains constant during drying ($N_u = 1$).

5. CONCLUSIONS

Based on a physical description of the conversion of bound and free water inside particles during drying, a new distributed model with parameters which all have a physical meaning and interpretation is presented, that describes the drying of materials. Due to their physical meaning and interpretation the parameters are known or can be estimated from experiments while the model can be adjusted relatively easy to other materials and dryers.

Optimal control computations illustrate the significance and feasibility of this model to optimize drying processes. The improved operation appears from an increased value of the performance index in [fl.kg^{-1}.s^{-1}] by protecting quality and saving energy costs. Moreover the results indicate the importance to optimize the drying time. Actually, spatial quality distribution aspects should be considered in the optimization procedure, since product quality is characterized by internal profiles (Quirijns, 1998; Quirijns et al., 1998). However, in order to judge the feasibility of the model, here commonly applied average quality aspects have been considered.

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In this paper the physical model parameters that need to be estimated have been obtained from literature describing, as much as possible, comparable drying circumstances and materials. Despite the physical meaning and interpretation of the model parameters, the model needs validation against experiments. These experiments will also be used to estimate those model parameters which could not reliably be obtained from the literature.

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APPENDIX 1: NOMENCLATURE AND PARAMETERS

a_w = water activity

C_{aw} = constant in GAB sorption equation, 28.

c_{pa} = heat capacity of air [J.kg^{-1}.K^{-1}], 1010.

$c_{p,dm}$ = heat capacity of dry material [J.kg^{-1}.K^{-1}], 1031.5

c_{pv} = specific heat of vapour [J.kg^{-1}.K^{-1}],1870.

c_{pw} = specific heat of water [J.kg^{-1}.K^{-1}], 4180.

c_1 = constant in penalizing function *(24.2)*, 1e4

$I\!\!D$ = internal diff. coeff. of moist. in material [m^2.s^{-1}]

$I\!\!D_0$ = diff. par. in Arrhenius-type rel. [m^2.s^{-1}], 6.64e-7

E_a = activ. energy for moisture diff. [J.mol^{-1}], 1.55e4

h = ext. convective heat transf. coeff. [J.s^{-1}.m^{-2}.K^{-1}], 97.

h_m = specific enthalpy of material [J.kg^{-1}]

J = cash flow [fl.kg^{-1}.s^{-1}]

J_D = energy costs per unit product [fl.kg^{-1}]

J_{DX} = total value of performance index [fl.kg^{-1}.s^{-1}]

J_T = heat flux on surface of material [J.s^{-1}.m^{-2}]

J_X = value of penalty function [fl.kg^{-1}.s^{-1}]

J_{XS} = mass flux, on surface of material [kg.m^{-2}.s^{-1}]

K_{aw} = constant in GAB sorption equation, 0.859

K_e = equilibrium constant in sorption process, 5.43

k = ext. convective mass transfer coeff. [m.s^{-1}], 0.09

k_a = adsorption rate constant [s^{-1}], 5.43e-4

k_d = desorption rate constant [s^{-1}], 1e-4

k_i = inactivation rate constant [s^{-1}]

M_a = molecular mass of air [kg.mol^{-1}], 0.02894

M_w = molecular mass of water [kg.mol^{-1}], 0.018

N = number of discretization points, 15

N_u = number of control intervals

P_{tot} = air pressure [Pa], 1e5

P_{sat} = saturation vapor pressure [Pa]

Q = residual activity [-]

$\mathcal{R}$ = gas constant [J.mol^{-1}.K^{-1}], 8.314

R = maximum radius of material [m], 5e-3

r = radius of material [m]

r_B = conversion rate bound/free water [s^{-1}]

T_a = air temperature [K]

T_{env} = temperature environment dryer [K], 293.

T_m = material temperature [K]

t = time [s]

t_f = total drying time [s]

t_r = reloading time [s], 600

X = moisture content [kg$_{H2O}$.kg$^{-1}_{ds}$]

X_B = concentration of bound water [kg$_{H2O}$.kg$^{-1}_{ds}$]

X_d = desired final moisture content [kg$_{H2O}$.kg$^{-1}_{ds}$]

X_F = concentration of free water [kg$_{H2O}$.kg$^{-1}_{ds}$]

X_m = monolayer value [kg$_{H2O}$.kg$^{-1}_{ds}$], 7.94e-2

Y_a = humidity of air [kg$_{H2O}$.kg$^{-1}_{da}$], 3e-3

Y_{as} = air humidity on surface of material [kg$_{H2O}$.kg$^{-1}_{da}$]

ΔH_v = evaporation enthalpy of water [J.kg^{-1}], 2.4e6

ϕ = air flow into the dryer [m^3.s^{-1}], 3e-3

γ = constant in perform. index [fl.J^{-1}.kg^{-1}], 5.06e-6

η = constant in perform. index [fl. kg^{-1}], 110

λ_m = thermal conduct. of material [J.s^{-1}.m^{-1}.K^{-1}], 0.464

λ_X = composed parameter *(12)*

θ = fraction of sites occupied in sorption model

ρ_a = density of air [kg.m^{-3}], 1.2929

ρ_m = density of material [kg.m^{-3}], 1470

parameter values quality model :

$a1$=101.31, $a2$=253129.9, $b1$=34.06, $b2$=119744.9, $a1'$=5.01, $a2'$=20580.3, $b1'$=-8.68, $b2'$=-22745.2, p=-906.58, q=6.09